This is an agent-based model to study the effects of reflexivity in the way innovations spread in a social network. Specifically, we propose to endow agents with a lightweight cognitive mechanism to allow them to recognize an emergent adoption pattern in the system and change their behavior according to that awareness. Our purpose is to study the effects created by the inclusion of reflexivity in the system. We must note that our model is a modified and extended version of a model previously developed by Delre et al. (2007) (our additions to their model will be made clear below).

At the beginning, agents are placed in the nodes of different kinds of social networks —scale-free, small world and random ones— and a small proportion δ of the them is specified as adopters of a new product introduced in the system. Then, at each time step, a non-adopter decides to adopt this product if she comes into contact with another adopter, and either her personal utility is greater than a certain minimal utility or she has been persuaded to adopt because of marketing. If D_i is the decision of agent i to become an adopter, then

$$D_i = \begin{cases} 1, & U_i \geq U_{i,\min} \text{ or } \lambda > s_i \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where U_i is her current utility, $U_{i,\min}$ her minimal utility, s_i her susceptibility to marketing and λ a constant that quantifies the amount of effort that goes into marketing. $U_{i,\min}$ and s_i are drawn from a uniform distribution $\mathcal{U}(0,1)$, with the first value being assigned to agent i before the simulation starts and the second one every time i is about to take her decision.

We consider that U_i depends on two kinds of social influences. First we have a local influence, which determines how useful it is for an agent to adopt given two factors: the rate of adoption of her closest neighbors and her individual preference[1]. If we call the utility derived from this influence as U_{Li}, we have that

$$U_{Li} = \beta \cdot x_i + (1 - \beta) \cdot y_i \quad (2)$$

$$x_i = \begin{cases} 1, & A_i \geq h_i \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$y_i = \begin{cases} 1, & p_i \leq q \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where β is called the coefficient of social influence, and it weights the importance of an agent's peers on her decision; A_i is the fraction of adopters among her closest neighbors[2]; h_i is the minimal fraction of adopters among those neighbors necessary to arise the desire to adopt; p_i is her individual preference, and q is the quality of the product she wants to adopt. Both h_i and p_i vary uniformly between 0 and 1, and they are set at the beginning

[1] This term accounts for the fact that an agent can decide that the product suits her personal needs even if their friends have not adopted it yet.

[2] If a node has no neighbors, this value is zero.

of the the simulation for each agent. Whereas β and q are global parameters of the model that take values between 0 and 1.

It is important to mention that the original model of Delre et al. (2007) goes up to this point. In other words, it describes diffusion of innovations as a process driven only by local influence, through equations 2, 3 and 4. The rest of this section corresponds to our additions to that model.

Besides local influence, our model also incorporates a global influence, which leads agents to adopt when they notice the appearance of a sizable portion of adopters in the population, even if they can not perceive a significant change in their surroundings. The simplest measure of the current amount of adoption in the system is the percentage of adopters. However, that does not take into account that agents are placed in a social network. In other words, the percentage disregards the structure of social relations that arise among adopters during the diffusion process, which depends on the underlying network topology. To overcome this limitation, we decided to use instead the average size of connected components in the subgraph of adopters. These components —called components of adopters from now on, for simplicity— correspond to subgraphs composed entirely of adopters and in which any two of them are connected by a path.

As this average size becomes bigger, the more useful it should be for an agent to join the trend and become one more of the crowd. Therefore, we define global utility in our model as

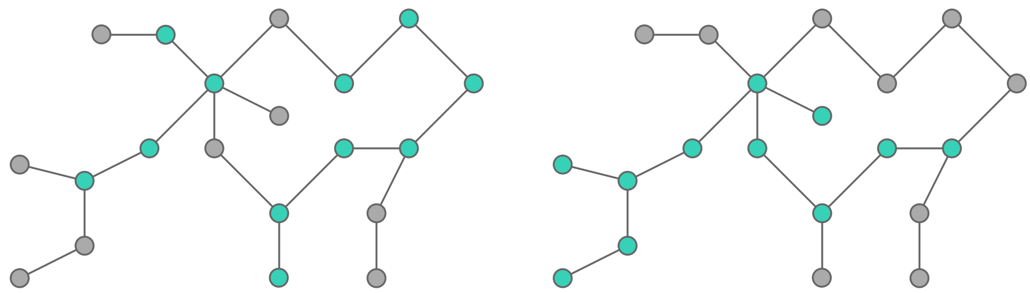
$$U_G = \frac{\overline{C}}{N} \tag{5}$$

$$\overline{C} = \sum_{j=1}^{n_c} \left(\frac{n_j}{N}\right) n_j, \quad n_j > 1 \tag{6}$$

where $\overline{C}$ is the (weighted) average size of components of adopters (cf. Fleiss et al., 2003, p. 441), N is the total number of agents, n_c is the number of components at time t, and n_j is the number of adopters in component j[3]. We do not take into account components of size one in equation 6 because we assume that agents do not acknowledge single individuals as categories of adopters (or non-adopters); that is, a focal agent begins to realize the existence of categories when groups of two or more connected adopters (or non-adopters) appear in the system. These categories of individuals do not necessarily need to be linked to the focal agent. As can be seen in Figure 1, $\overline{C}$ has the nice property of taking different values for different network configurations, even though the total number of adopters be the same in them.

When $\overline{C}$ is small, global utility also is, hence it should not play a part in agents' decisions. However, as $\overline{C}$ increases and gets closer to a certain critical mass, its effect should be felt more strongly and start influencing agents accordingly. To model this, we endow agents with a reflexive capacity to allow them to recognize that that critical mass has been reached or it is close to be reached. Only after becoming aware of that fact and being exposed to it for a certain amount of time, an agent can make use of U_G as part of her decision strategy.

[3]Note that U_G can only take values between 0 and 1 due to the way it is defined. This will be important in the definition of personal utility below.



(a) (b)

Figure 1: Average size of components of adopters $\overline{C}$ for two different network configurations with the same number of adopters (adopters are highlighted in green). (a) With 20 agents in total and two components of adopters of sizes four and seven, we have $\overline{C} = \frac{4^2+7^2}{20} = 3.25$ (see equation 6). (b) With a single component of size eleven, we have $\overline{C} = \frac{11^2}{20} = 6.05$ (i.e. almost twice the one for the previous configuration).

Specifically, we assign agents at the beginning of the simulation a reflexivity index $\alpha_i \sim \mathcal{U}(0,1)$, to account for heterogeneity in their reflexive abilities. During each time step, we compare this index to an emergence factor E, that increases in value (from 0 to 1) as the global utility approaches the critical mass. We define this factor through the following logistic equation

$$E(U_G) = \frac{1}{1+e^{-\phi(U_G - M_c)}} \tag{7}$$

Here M_c is called the critical mass and corresponds to the fraction of adopters in connected components needed for agents to regard that an emergent adoption pattern has appeared in the system. ϕ, on the other hand, controls how sharp the transition is from not detecting that the system has reached M_c to actually doing it. Both M_c and ϕ are global parameters, with M_c limited to have values between 0 and 1. We consider an agent becomes aware of the appearance of M_c when the condition $E(U_G) > \alpha_i$ is reached. Figure 2 displays a plot of equation 7 and its relationship to α_i.

Finally, agents in our model do not start using the knowledge gained through reflexivity immediately after becoming aware of a global pattern. Instead, we record the amount of time that has passed since each agent detected M_c. Only when that time is higher than a personal delay threshold, they can use global utility for their decisions. We obtained this idea from generalized models of contagion (Dodds & Watts, 2005, 2004). In these models agents receive one dose of the contagious entity (e.g. a disease or rumor) per time step, and an agent becomes infected when the amount of doses surpasses a threshold. In our case, we use this concept to model that agents need to be exposed to the perception of an emergent adoption trend for a certain period before it can have an effect on them. This seeks to capture the fact that people responses occur at different time scales because there are several psychological factors (e.g. feelings and willingness to act) that influence their decision process (Sornette, 2006).

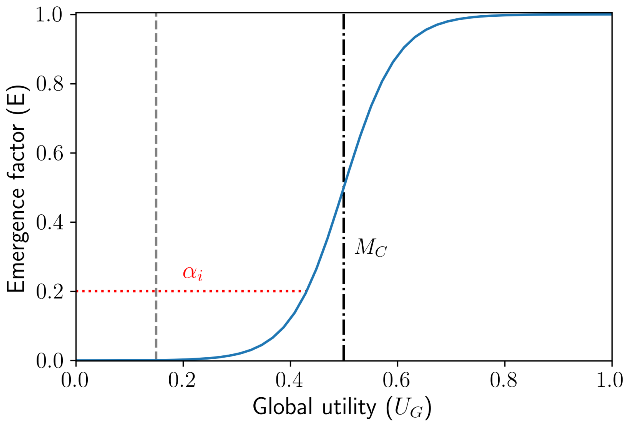


Figure 2: Plot of the emergence factor E as a function of the global utility (see equation 7), along with agent's i reflexivity index α_i and the critical mass of adopters in components M_c. As an example, the figure shows a value of $\alpha_i = 0.2$, above which the emergence $E(U_G)$ will be taken into account by the agent in her decision to adopt. The dashed gray line corresponds to the first value of U_G for which $E > 1 \times 10^{-5}$. We have arbitrarily decided that this value of U_G sets the instant from which agents can make use of global utility in their decisions.

Given equations 2, 5 and 7, we define personal utility U_i as

$$U_i = \begin{cases} U_{Li} + U_G - U_{Li} \cdot U_G, & E\left(U_G\right) > \alpha_i \text{ and } t_a > d_i \\ U_{Li}, & \text{otherwise} \end{cases} \tag{8}$$

where t_a is the time elapsed since agent i realizes the appearance of M_c and d_i is her delay threshold before including that awareness in her utility. As can be seen, equation 8 reflects that when agents detect emergence due to their reflexive capacity and enough time has passed to be influenced by that information, their utility depends on the disjunction of U_{Li} and U_G[4]. In other words, at that point they decide to adopt according to their most preponderant utility, which can be either local or global. Before that agents' decisions are governed by local factors only.

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[4] Since U_{Li} and U_G vary between 0 and 1, the first part of equation 8 corresponds to a disjunction in probability theory.

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