1 Overview

1.1 purpose and patterns

The model is suitable to investigate the effects of different characteristics of apprenticeship programmes both in historical (old Britain, Armenian merchants of New-Julfa, and the British East India Company) and contemporary societies (German and contemporary Britain). The model is built considering five societies , using an agent-based simulation model, we identified six main characteristics which impact the success of an apprenticeship programme in a society, which we measured by considering three parameters, namely the number of skilled agents produced by the apprenticeships, programme completion, and the contribution of programmes in the Gross Domestic Income (GDI) of the society. We investigate different definitions for success of an apprenticeship and some hypothetical societies to test some common beliefs about apprenticeships performance. The model also shows the number of unemployed agents given their work-based skills, wages, and the number of small and large companies who participate in training agents. The model enables exploring the impact of parameters, such as initial wages and the number of training years, along with the stated policies on the system.

1.1.1 Purpose

The identification and systematic modelling of the impact of characteristics and institutions (i.e. "the rules of the game," such as restrictions imposed by guilds; North (1990)) on the performance of apprenticeship programmes, is addressed. Therefore the key questions are (a) which one of them is more important and (b) why different societies do not follow the same approach. The main purpose of this study is to address these concerns, and we employ agent-based simulation to improve our understanding of the mechanics that made some apprenticeship programmes in completely different times and locations, such as the Julfan and the German, more efficient in terms of improving societal skill level and the GDI.

1.1.2 Patterns

We evaluate our model by its ability for reproducing six patterns as follows:

Pattern 1 and 2; the impact of parameters on the training quality of small companies (or contractors) and large companies who participate in programme. In the societies studied we identify the impact of restrictions on the number of eligible trainers (e.g. guilds and unions) and trained apprentices on the quality of training. Besides stated restrictions, other characteristics such as openness of the society, and skills influx from other societies, engagement of schools to train agents impact the intention and quality of training by trainers. This pattern indicates how using such institutions help societies to have a stable number of trainers.

Pattern 3; unemployment of trained apprentices. In certain societies (e.g. old Britain) it is observed that there were some unemployed trained apprentices that had negative impact on the participation of both trainers and trainees in the programme. This pattern indicates how some restrictions (e.g. guilds and unions) help to control this unemployment. Also, openness of the society impact this as observed in open society, because some agents may leave the society to work elsewhere.

Pattern 4; wages. This indicates how the apprenticeship programme helps controlling wages. A reason for trainers (especially for manufacturers) is to keep wages as low as possible; this way their human resource costs will decrease. However, for an artisans low wage is a negative point (it is their source of income); therefore they may prefer to train less agents so that they will be able to keep their income high. Different behaviours are reported for each trade type that can be replicated by this model.

Pattern 5: predicted Gross Domestic Income (GDI). This indicates the impact of apprenticeship programmes at the societal level. For instance, New-Julfa had flourished in less than a century because of its merchants' higher trading skills than other contemporaneous merchants.

Pattern 6: completion ratio. This shows the ratio of apprentices who successfully finished apprenticeship programmes. Old Britain, as an instance, had a low completion ratio because of its openness (e.g. trainees left London to work in smaller cities wherein demand for low-skilled agents was available) and the quality of training offered.

Pattern 7: Skills. This shows the number of skilled agents. Old Britain, as an instance, had a weakness because of some of the quality of training offered.

1.2 Entities, state variables, and scales

The following entities are available in the system:

Potential apprentices: These are agents who might be chosen to be trained by trainers. These agents represent the part of the society who are interested in learn a skill to work as an artisan (e.g. goldsmith) or for a manufacturer (e.g. maintenance expert).

Trainers: These are agents who train agents. The trainers are decided based on the trade types as follows:

Artisans: An artisan, such as a wood-carver or a hairdresser, trains a potential competitor but can employ the trainee's services during the apprenticeship. For these reasons, in some societies, artisans consider those services insufficient for compensating for the costs, and ask for some payments from apprentices. In addition, some artisans use guilds as a means of restricting trained agents from getting into the system to guarantee the stability of wages in the future. These trades do not require complicated analytical skills or computations, etc. that are provided by school-based training, and the artisans are the only ones who are eligible to train others (i.e. contractors cannot replace the artisans).

Manufacturer: Alternatively, a trader's or manufacturer's trainee does not compete with apprenticeship providers, unlike a trained artisan. For instance, setting up a factory or starting long-distance trades require a large amount of capital when compared to a salon or to buying tools and working at home. Therefore a trained apprentice is a potential employee (not a competitor as in the case of an artisan).

Manufacturers have future benefits in training apprentices. Therefore companies train agents without asking agents for prepayment. These benefits include negotiating for paying slightly lower wages than community norms, hiring the best-trained agents, and providing training in some specific skills required for that particular manufacturing company. Note that these trades require complex knowledge and analytical skills that may require school-based training. Furthermore, these skills can also be transferred by employing third-party trainers (i.e. contractors).

What follows lists the parameters used in the simulation, along with their explanations:

Parameter name	Explanation
AcademicSkillThreshold	The threshold for academic skills beyond which an agent's skills is useful
AdequateWBSkillThreshold	The threshold for work-based skills beyond which an agent's skills is useful
alphaMu	The average for an agent's discount factor for money
ArtisanWagesStickinessThreshold	The threshold for stickiness of wages for artisans

Parameter name	Explanation
BadTrainingOnSpeed	The impact of bad training on an agent's speed of learning
Check	The probability by which an agent considers leaving the programme with its current skills
ExcellentWBSkillThreshold	The threshold for academic skills beyond which agent's skills is useful
Guild-restrictions?	The threshold for work-based skills beyond which an agent's skills is excellent
Horizon	The number of years that an agent considers in its calculation of its utility function
ImpactOfSchoolOnSkill	The impact of attending schools on an agent's speed of learning of work-based skills
Influx	Determines whether the system accepts skilled immigrants
IWAd	Initial wages for agents with adequate skills (Artisans)
IWEdAd	Initial wages for uneducated agents with adequate skills (manufacturers)
IWEdEx	Initial wages for educated agents with excellent skills (manufacturers)
IWEx	Initial wages for agents with excellent skills (Artisans)
IWNEdAd	Initial wages for uneducated agents with excellent skills (manufacturers)
IWNEdEx	Initial wages for uneducated agents with excellent skills (manufacturers)
LearningSpeedOverEstimation	The overestimation of an agent about its learning speed
NoPassionsImpactOnSpeed	Impact of lack of passion for leaning on speed of learning
NoUnion	Determines whether the system has an active union
number-of-big-companies	The number of larger companies and artisans with more sophisticated skills
NumberOfPotentialTrainees	The number of potential attendees from whom the trainers choose their apprentices
NumberOfSmallCompanies	The number of smaller companies and artisans with simpler skills
Open?	Determines whether the system accepts apprentices from outside the society
prepayment	Determines whether the trainers ask for a prepayment from their apprentices
ProfBE	Profit of producing an item by an agent with excellent skills in large companies (Artisans)
ProfBEdE	Profit of producing an item by an educated agent with excellent skills in large companies (Manufacturers)
ProfBEdG	Profit of producing an item by an educated agent with adequate skills in large companies (Manufacturers)
ProfBG	Profit of producing an item by an agent with adequate skills in large companies (Artisans)
ProfBNedE	Profit of producing an item by an uneducated agent with excellent skills in large companies (Manufacturers)
ProfBNedG	Profit of producing an item by an uneducated agent with adequate skills in large companies (Manufacturers)
ProfSE	Profit of producing an item by an agent with excellent skills in small companies (Artisans)
ProfSEdE	Profit of producing an item by an educated agent with excellent skills in small companies (Manufacturers)
ProfSEdG	Profit of producing an item by an educated agent with adequate skills in small companies (Manufacturers)
ProfSG	Profit of producing an item by an agent with adequate skills in small companies (Artisans)
ProfSNedE	Profit of producing an item by an uneducated agent with excellent skills in small companies (Manufacturers)

Parameter name	Explanation
ProfSNedG	Profit of producing an item by an uneducated agent with adequate skills in small companies (Manufacturers)
school-or-knowledge	Determines whether the system uses schools to transfer educational skills to apprentices
ShareOfWealthyFamilies	The proportion of wealthy families who can pay for premiums in a society
SmallCompaniesCanTrain	The number of smaller companies that are allowed to train apprentices when all large companies participate
TraineesCurrentInformationWeight	The weight apprentices/ trainees give to the current information
TraineesMaxWaitingYears	Maximum years an unemployed agent search for a job
TrainersCurrentInformationWeight	The weight trainers give to the current information
Trainer-type	Trainer types are defined as follows: 0 - companies as potential employers, 1 - contractors, 2 - artisans
TrainingSpeedReductionForSmallCompanies/Contractors	Impact of being trained by lower-skilled agents on speed of learning
TrainingYears	The apprenticeship duration
WagesStickinessThreshold	The threshold for stickiness of wages for manufacturers

Note that scales of the model (e.g. the number of potential trainers) can be modified so that their impacts on different patterns can be tested.

1.3 Process overview and scheduling

The simulation model concept is split into three executive procedures. The first procedure is executed with the societal level set-up, including the creation of an appropriate society as artisans or a manufacturing society. The second procedure covers the decision-making of trainer agents. The third algorithm describes the procedure of individual apprentices. In each run, these procedures are executed in sequence. Note that all loops run once per iteration. Figure 1 illustrates how two agent types interact with each other and the decision variables external to them. The trainers compare the profit of training with other alternatives (i.e. they take account of already trained agents and hiring from other communities or graduates). In parallel, agents decide whether or not to participate in the apprenticeships by taking account of demand for the skill, the cost of lost fortune over the training period, and their ability to pay fees. If two agent types decide to participate, based on the rules (e.g. restrictions on the number of trainers) and capacity of trainers, the apprenticeship begins. In each iteration the system parameters are updated, trainees may revise their decision, and new potential trainers are introduced into the system. The following paragraphs present a detail of these decision procedures (see the paper for an explanation of algorithms).

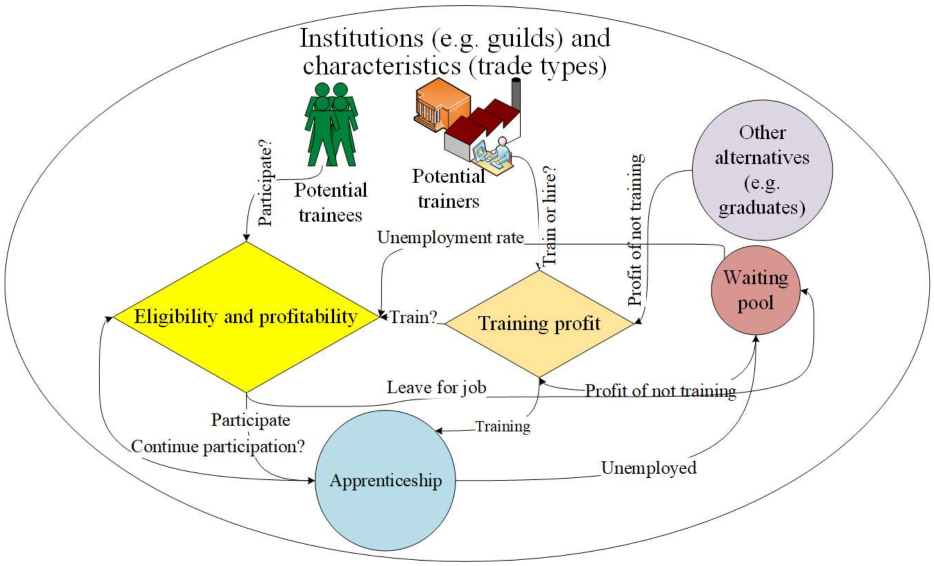

Figure 1 Interaction of different agent types (trainers and trainees) with each other and with the external decision variables.

2 Design concepts

2.1 Basic principles

The model investigates how different apprenticeship programmes, along with other parameters, impact the overall wages, skills, Programme completion ratio, and Societal prosperity (GDI). For instance, we know certain programmes were more successful in increasing societal skills. Such an investigation helps policy makers to test different apprenticeship programmes in an artificial society to make the best decision.

2.2 Emergence

Main outcomes of the simulation include, the number of skilled agents, the GDI by apprenticeship programmes, and the completion ratio over time. These findings help the policy makers to make better decisions based on their intentions. For instance, a society may need a short-term increase in the number of skilled agents while another society has a long-term policy for improving their GDI. Using the produced patterns, one can observe some counter-intuitive patterns such as negative correlation between the completion ratio and skills of the society.

2.3 Adaptation

Overall, agents, given their types, have different adaptative behaviour. Potential apprentices, decide whether or not participate in the programme or continue their participation. When an apprentice decides to participate, it checks a) if it can afford to pay costs (if there is any) and b) if it is profitable to be trained if a trainer accept them. During training period, apprentices decide if they want to leave the programme or wait until the completion. Trainers decide if they want to recruit new agents or skip this year and they also decide about the quality of training.

2.4 Objectives

The objective that decisions are made upon is the predicted utility function in terms of income/ profits. The agents make a decision that maximises their utility function.

2.5 Learning

The agents' skills and the trainers' current policy are numbers that can be assessed by asking former trainers and references in both contemporary and historical instances. Agents discount past information using a weight for long past and recent information.

2.6 Prediciton

The adaptive behaviour of agents is based on their predicted utility function. In other words, they take an action that maximises their predicted utility function.

2.7 Interaction

The interaction of agents happens through training. An apprentice interacts with its trainer. Overall, this interaction impacts the maximum skill level of a trainee (through transferring skill and the quality of training) and also the utility made for a trainer (e.g. s company hires a high skilled agent).

2.8 Stochasticity

The model is initialised stochasticly. Agents' attainability, discount factors, and passions are randomised. Also, for open systems, in each run some agents randomly leave the society.

2.9 Initialization

In the simulation, we used parameter values shown in Table 3 of the paper (see Appendices B and C for an explanation of chosen values and a sensitivity analysis, respectively). The simulation was initialised with random values for probabilities to be learnt by the agents. Also, row numbers 9 and 10 of Table 3 shows the initial wages that the simulation was started with. Some of the simulation constants (e.g. Pareto principle) are chosen based on empirical studies (see Appendix B for a description of chosen values). Other simulation constants are chosen such that they reflect our information about those societies (e.g. sustainability of apprenticeship programmes). We have performed a sensitivity analysis to test the impact of these values in Appendix C.

3 Submodels

we discuss how different agent types make decisions by discussing their utility functions. First we discuss the utility function and decision procedure associated with trainees.

Trainees have two distinctive policies, identified by $\pi_{Trainees}(policy)$, namely not to attend (NA) and attend the programme (A). The expected utility function associated with not attending programme over y years' decision horizon, tr years of training, and considering a discount factor of α, W_l unit income per year for skill l and a prepayment of $premium$ is calculated as follows:

$$\pi_{Trainees}(NA) = \sum_{t=1}^{y} \alpha^t \times W_l,$$

$$\pi_{Trainees}(A) = \sum_{\forall l} \sum_{t=tr+1}^{y} P_h \times P_l \times \alpha^t \times W_l - Premium + IncomeTrainer(t) \times \alpha^t,$$

where, P_l is the probability of acquiring skill l, and P_h is the probability of getting hired. The income of being a trainer ($IncomeT\ rainer(t)$) at time t (for artisans) is calculated as follows:

$$IncomeTrainer(t) = (1 - P_{Tra}^t) \times premium,$$

where $(1 - P_{Tra}^t)$ is the probability of being a trainer after finishing a programme, and is an increasing function of their experience. The agents who attend the programme, decide whether to continue (C) or not (NC). The following provides the expected utility of trainees at year x of training.

$$\pi_{Trainees}(C) = \sum_{t=tr+1-x}^{y} Pr_{Es} \times \alpha^t \times W_{Es},$$

$$\pi_{Trainees}(NC) = \sum_{t=1}^{y} Pr_s \times \alpha^t \times W_s,$$

where, E_s and s are expected acquired skill and acquired skill. Also, Pr_{Skill} and W_{Skill} are the probabilities of
finding a job and the wages associated with a skill ($Skill$). Note that during apprenticeship (i.e. before tr −x),
trainees who continue the programme (i.e. when its policy equals C) cannot improve their utility function.
On the other hand, the trainers have three policies to improve their utility functions$\pi_{Trainers}(policy)$, namely
good (G), bad (B), and avoid training (N). Overall, each trainer considers cost of training (e.g. competition and
training costs), and profits of training (e.g. paid subsidies and prepayments) based on its type. In the following, we show utility functions associated with different training policies for companies:

$$\pi_{companies}(G) = \sum_{\forall l} \sum_{t=tr+1}^{y} Pr_{G,l} \times (profit(l) - W_{G,l}) \times \alpha^t - n \times \sum_{t=1}^{tr} Cost \times \alpha^t,$$

$$\pi_{companies}(N) = \sum_{\forall l} \sum_{t=1}^{y} \{Pr_{N,l} \times (profit(l) - W_l)\alpha^t,$$

wherein, $Pr_{G,l}$ and $Pr_{N,l}$ are the probabilities of finding an agent with skill l under good and no training policies. As discussed in the paper we assume that companies pay wages lower than norm (i.e. $W_{G,l} < W_l$). Finally, $profit(l)$ is profits obtained by companies having an agent with skill l, and companies consider the number of trainees (n) and costs of training over the training period (tr).
Utility functions associated with different training policies for contractors are as follows:

$$\pi_{contractors}(G) = Pr_{G,Hired} \times subsidy(Hired) + cost(G),$$

$$\pi_{contractors}(B) = Pr_{B,Hired} \times subsidy(Hired) + cost(B),$$

where, $Pr_{G,Hired}$ and $Pr_{B,Hired}$ are probabilities that a trainee finds a job under good and bad training policies, respectively. Also, $subsidy(Hired)$ is a paid subsidy per agent who finds a job. Finally, $cost(G)$ and $cost(B)$ are costs of good and bad training. Note that considering paid subsidies for training, costs of bad training are negative (i.e. trainers make some profit).
Utility functions associated with different training policies for artisans are as follows:

$$\pi_{artisans}(G) = Prepayment + \sum_{t=1}^{tr}(Wage_{labour} \times \alpha^t) - \sum_{t=1}^{y}(Pr_{G,ExcessSupply} \times \alpha^t),$$

$$\pi_{artisans}(B) = Prepayment + \sum_{t=1}^{tr}(Wage_{labour} \times \alpha^t) - \sum_{t=1}^{y}(Pr_{B,ExcessSupply} \times \alpha^t),$$

where, $Prepayment$ is the paid costs for training, and $Wage_{labour}$ indicates the labourer works done by apprentices during the programme. Also, $\mathrm{Pr}_{G,ExcessSupply}$ and $\mathrm{Pr}_{B,ExcessSupply}$ are probabilities of a decrease in wages for excessive labour supply under good and bad training policies, respectively. Finally, not training produces a utility function with a value of zero.