

# Model description

## 1 Purpose

Axelrod[1] demonstrated that meta-norms are necessary to maintain norms by modeling from an evolutionary approach and simulation in 100 generations.

In this simulation, we modify the norms game model to bid-rigging (collusion) model, while we can simulate also the Axelrod's norms game model.

## 2 Model rules

### 2.1 Axelrod's norms game model

Axelrod's norms game model is as follows: The strategy of an agent has two dimensions. The first is boldness ( $b_i$ ), which determines when the agent will defect. The agent will defect whenever the chance of being seen by someone is less than the agent's boldness, which is represented as  $S < b_i$ . The second is vengefulness ( $v_i$ ), which is the probability of punishing defecting agents. The greater an agent's vengefulness is, the more likely he or she will punish defecting agents.

Axelrod[1] implemented an evolutionary approach to the norms game model. For each generation consisting of four games, we calculate the standard deviation of the utility in that generation and replace the strategies of agents whose utility is less than the standard deviation with strategies of agents whose utility is greater than the standard deviation. In the model of Axelrod[1], to adjust the number of offspring so that the size of the population  $n$  does not change between generations, but the method is not shown. Therefore, We use the method (adjustment 2) shown by Yamamoto and Okada[3], i.e., randomly moving agents selected arbitrarily from groups within the standard deviation to the smaller of the two groups above and below the standard deviation.

Then, following Axelrod[1], we allow the strategy to mutate with a probability of 1percent.

## 2.2 Bid-rigging norms game model

McAfee and McMillan[2] studied rigging in auctions. They characterized coordinated bidding strategies in two cases. Weak cartels, where bidders cannot make side-payments; and strong cartels, where cartel members can exclude new entrants and make transfer payments.

In this simulation, we can change the model, "strong" or "weak".

Each agent has two strategies, boldness ( $b_i$ ) and vengeance ( $v_i$ ), and the two strategies are random variables ranging from  $\frac{0}{7}$  to  $\frac{7}{7}$ . First, each agent decides whether to betray or cooperate with the collusion mechanism according to its own boldness  $B_i$ . If "all  $n$ " participating agents cooperate, the collusion mechanism splits the auction (bidding) winner's gain  $R$  among all ( $\frac{R}{n}$ ). If there are more than one betraying agent, there will be competition among the betraying agents, and the probability that an individual betraying agent wins the bid is  $\frac{1}{n_D}$ , where  $n_D$  is the number of betraying agents, and with this probability we get a betrayal gain of  $T$ . Then, the betraying agent is always discovered and the other agents punish the betraying agent  $P$  with the probability of revenge ( $v_i$ ) and with the punishment cost  $E$ .

Then, following Axelrod[1], we implement the evolutionary approach, and allow the strategy to mutate with a probability of 1percent.

Fig. 1 depicts the bid-rigging norms model. In this simulation,

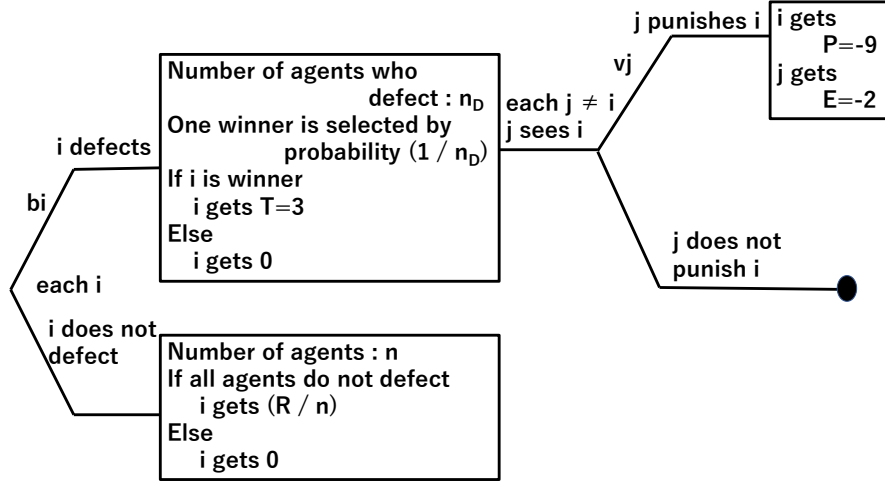


Figure 1: The "Bid Rigging Norms" Model

We change the norms game model at the four points as follows:

1. We introduce reward( $R$ ) in this model. In Axelrod's[?] norms game model, agents who cooperate will get  $R = 0$  (Fig. ??). However in the bid-rigging norms, we should assume that agents who cooperate will get  $R > 0$ , while  $R > T$ . If all agents cooperate (collude) in strong cartels, all agents can get  $R$ , which is divided among all agents (Fig. 1).
2. Only one of the defecting agents can get  $T$  because one winner is selected in auctions, whereas, in Axelrod's[?] model, all defecting agents can get  $T$  by the exploitation of public goods.
3. Hurt( $H$ ) is omitted because we cannot assume "hurt" to other agents by defecting in the bid-rigging norms model, whereas "hurt" make snense in Axelrod's[?] norms game model because the model is one of the social norms models for public goods.
4.  $S$ , which represents the probability of being observed or seen, is omitted because the identity information and the bid of the winning agent are public.

### 3 Parameters

We can set the parameters' value on the interface. The parameters which we can set on the interface is listed at Table1.

### References

- [1] Robert Axelrod. Promotig norms. In *The Complexity of Cooperation*. Princeton University Press, 1997.
- [2] R Preston McAfee and John McMillan. Bidding rings. *American Economic Review*, 82(3):579–599, 1992.
- [3] Hitoshi Yamamoto and Isamu Okada. Evolution of cooperation by a social vaccine. *The IEICE transactions on information and systems (Japanese edition)*, 94(11):1836–1846, 2011.

Variable	Description
model-change	We select game models from "Axelrod", "collusion".
model-change-2	We select the bid-rigging game models from "strong", "weak".
S-rate-change	We select "on" in the case of "axelrod", "off" in the case of "collusion".
meta-norm-game	We select "on" in the case of "axelrod", "off" in the case of "collusion".
data-discharge-per-steps	We select "on" if we need to discharge data for analysis.
mutation-rate	Agents mutate their strategies by "mutation-rate". It is 0.01, following Axelrod.
length-of-run	The simulation is run for this number of generations. It is 100, following Axelrod.
number	The number of agents. It is 20, following Axelrod.
UF	The value of "Reword"
UM	The value of damages to other agents in the case that an arbitratry agent cooperate.
UT	The value of "Temptation". It is 3, following Axelrod.
UH	The value of "Hurt". It is -1, following Axelrod.
UE	The value of "Enforcement". It is -2, following Axelrod.
UP	The value of "Punishment". It is -9, following Axelrod.
initial-B-rate	We set the numerator of boldness from 0 to 7.
initial-V-rate	We set the numerator ofvengefulness from 0 to 7.

Table 1: Parameters on the interface