The Price Evolution with Expectations Model

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Model Purpose

The Price Evolution with Expectations model provides the opportunity to explore the question of non-equilibrium market dynamics, and how and under which conditions an economic system converges to the classically defined economic equilibrium. To accomplish this, we bring together two points of view of the economy; the classical perspective of general equilibrium theory and an evolutionary perspective, in which the current development of the economic system determines the possibilities for further evolution.

Given independent demand and perfect competition, classical economics dictates there exists a price vector for goods in an economy such that all markets clear. Simplified, given an aggregate firm and household, and a single representative good, there is a single state described by good quantity, labour and price, where all labour is used and all of the good sold. Theoretically, this vector is assumed to be discovered by a Walrasian auctioneer, but we take a different approach where the firm learns the household demand over time and the system evolves accordingly.

The Price Evolution with Expectations model consists of a representative firm producing no profit but producing a single good, which we call sugar, and a representative household which provides labour to the firm and purchases sugar.The model explores the evolutionary dynamics whereby the firm does not initially know the household demand but eventually this demand and thus the correct price for sugar given the household's optimal labour.

The model can be run in one of two ways; the first does not include money and the second uses money such that the firm and/or the household have an endowment that can be spent or saved. In either case, the household has preferences for leisure and consumption and a demand function relating sugar and price, and the firm has a production function and learns the household demand over a set number of time steps using either an endogenous or exogenous learning algorithm. The resulting equilibria, or fixed points of the system, may or may not match the classical economic equilibrium.

The model is written in Python3 with an Object Oriented structure, and is based on the work described in Global Climate Forum Working Paper 3/2016 by Steudle, Yang and Jaeger. [1]

[1] https://globalclimateforum.org/2016/12/01/price-dynamics-via-expectations-and-the-role-of-money-therein/.

Model Overview

The Classical View

The representative household intends to purchase sugar, S, according to price, p. The household demand function is

$$S = ap^b \tag{1}$$

where $b < 0$, $a > 0$, and both p and S are positive real values. Thus b can be understood as the price elasticity of demand for our composite good, sugar.

The household also provides labour, L, according to maximizing a simple Cobb-Douglas utility function for sugar and leisure, $L_{max} - L$, such that

$$U(L, S) = (L_{max} - L)^{\alpha} S^{\beta}, \tag{2}$$

where α is the household preference for leisure and β the preference for sugar. In simple Cobb Douglas terms, $\alpha + \beta = 1$.

The firm has a simple production function where a certain amount of sugar is produced by a given amount of labor, L, which is

$$S = L^{\gamma}. \tag{3}$$

The aggregate firm will make no profit, and given wage as numéraire with $\omega = 1$, this means the firm will choose labour such that

$$\pi = L^{\gamma} * p - L = 0. \tag{4}$$

The household provides labour to the firm, and the amount provided is the minimum of the cost of the desired sugar at the advertised price and the amount that maximises the household utility. This means the household won't work more than the utility maximising amount if the price of sugar is high in order to consume the utility maximising sugar, rather the household will forgo sugar rather than working more than the utility maximising value. Conversely, if the household can afford its utility maximising amount of sugar by working less than the utility maximising number of hours, they will do so.

The firm then employs the minimum of the labor provided by the household and the labor it can afford toward producing sugar. The household provides the agreed labor, the firm produces the sugar it can with the labor provided. The household then purchases the quantity of sugar at the firm's price either according to what it can afford or its demand at that price, whichever is less.

The firm uses the revenue from the previous market interaction to purchase labor for the next production cycle. Revenues from the previous cycle thus become the budget constraint for the next cycle.

Given no endowments and $\omega = 1$, the budget condition is such that L is available to purchase S. With price of sugar given by p and using the substitution method with $S = L/p$, we find $L^{\star}$ by

solving

$$L^\star = \max_L \quad (L_{max} - L)^\alpha \left(\frac{L}{p}\right)^\beta. \tag{5}$$

For our Cobb-Douglas preference function $\alpha + \beta = 1$, the solution is

$$L^\star = \beta L_{max}. \tag{6}$$

Note that $L^\star$ is not a function of p.

In the equilibrium condition with perfect competition, there is no firm profit so $p^\star S^\star - L^\star = 0$ and the equilibrium price is

$$p^\star = (\beta L_{max})^{1-\gamma}. \tag{7}$$

At equilibrium, the point $(p^\star, S^\star)$ must fall on the household's demand function such that

$$S^\star = a(p^\star)^b. \tag{8}$$

Given b,

$$a = \frac{S^\star}{(p^\star)^b}. \tag{9}$$

The household demand function is also consistent with the Cobb-Douglas preferences in the case of unitary elasticity of substitution, or $b = -1$. To see this, we maximise the utility function with respect to sugar, substituting the budget condition $L = pS$ for labour in Equation 2. We find

$$S = \frac{\beta L_{max}}{p} \tag{10}$$

and comparing with Equation 1 with $b = -1$, we find

$$a = \beta L_{max}. \tag{11}$$

The firm represents an aggregate firm, and at equilibrium the profit of the aggregate firm, $\pi(L)$, will be zero. Thus the firm will choose L_{opt} such that $pS = \omega L_{opt}$, or in the case of $\omega = 1$, $pS = L_{opt}$. The firm has an expectation of the demand for sugar at a given price, p, where the demand function is of the same form as that for the household, $S = ap^b$, such that

$$p = \left(\frac{S}{a}\right)^{\frac{1}{b}}. \tag{12}$$

Sugar production is a function of labor such that $S = L^\gamma$, thus the profit formula, $\pi(L)$, can be expressed solely in terms of L and the model parameters

$$\pi(L) = L^\gamma \left(\frac{L^\gamma}{a}\right)^{\frac{1}{b}} - L. \tag{13}$$

We can explicitly solve for the L_{opt} that produces zero profit, which is

$$L_{opt} = (\beta L_{max})^{\frac{1}{(\gamma+\hat{b}\gamma-\hat{b})}}. \tag{14}$$

and with $b = -1$ we find $L_{opt} = \beta L_{max}$ which is the same as $L^\star$.

Thus the system is consistent with the household demand and utility functions, the firm production function and the zero-profit condition. The starred values, $p^\star, S^\star$ and $L^\star$, represent the classically defined economic equilibrium state.

An Agent Based Model Adds Some Realism to the Classical View

The conventional view on price setting is that there is an all-knowing auctioneer that adjusts quantities and prices instantaneously such that the economic equilibrium is obtained. This entity is a fiction. More importantly, the firm will have an expectation of the households demand which may or may not be correct, and will take some time to learn the true demand by observing the realised (S, p) combinations.

We have constructed a model as simple as possible for exploring the evolution of the system where expectations of price differ. In brief:

- The agents are an aggregate firm that produces an aggregate good called sugar from labour, and an aggregate household that supplies labour and consumes sugar.
- The agents have preferences and decision rules that determine their supply and demand of labour and sugar. The aggregate firm, as a representative firm in equilibrium with perfect competition, minimises its profit and the aggregate household maximises its utility to determine supply and demand.
- The price setting agent has expectations about the relation of prices and aggregate supply and demand. Prices are set by the firm based on this expectation.
- The price setting agent updates its expectations using observations about the actual excess demand at the prices that have been set before.
- There are two markets, a labour market and a sugar market. In both markets, if supply does not equal demand, the short side of the market prevails.
- In every time step, first transactions take place in the labor market and later in the sugar market.
- In both markets, the buyer is limited by its budget and there is an upper limit of how much it can afford. If there is money, this upper limit is given by its cash balance. If there is no money, for the household (buyer in the sugar market) the upper limit is given by the value of labour it sold at the last transaction in the labour market. For the firm (buyer in the labour

market) the budget is given by the value of sugar it sold at the last transaction in the sugar market.

There are two versions of the model, one without money and one with what we will call "light" money:

1. No money version: There is no money, only sugar and labour of equal value can be exchanged quasi-instantaneously such that two transactions are executed one after another.This means the value of labour or sugar can be stored for the short period of time needed from the point of time transactions take place in one market to the point they take place in the other market.

2. Money version: Money exists as a fungible store of value, and in the beginning the firm and household may have a certain endowment of money at their disposal. However, for the household's utility optimisation money is irrelevant, as well as in the firm's profit minimisation. That means that there is no demand for money. Labour is still the numéraire and one unit of money always has the same value as one unit of labour.

The Firm Learns the Household Demand Over Time

To distinguish from the household's demand function $S = ap^b$, the firm's demand function is parameterised as

$$S = ap^{\hat{b}} \tag{15}$$

where $\hat{b}$ is the firm's expectation of the household value for b. The learning process describes the convergence of $\hat{b}$ with b. This convergence is implemented in two different ways: as exogenous learning and as endogenous learning.

Exogenous learning occurs over a given number of steps, where the total error in $\hat{b}$ compared to b is divided by the number of steps into equal segments, and each iteration of the model reduces that error by a single segment until $\hat{b} = b$.

With endogenous learning the firm receives a signal about household demand through the sugar market. In a model this simple, a single correct signal will immediately give the firm the correct value of b. We would like the convergence of $\hat{b}$ to b to occur over a number of time steps such that we can better observe the system's evolutionary dynamics. Therefore we implemented a learning process where a correct signal will pull be $\hat{b}$ toward b while an incorrect signal will not. Either way, the firm determines p and observes the household response, S. The firm then identifies b from this signal as

$$b_{obs} = \frac{\log(\frac{S}{a})}{\log p}. \tag{16}$$

If the convergence takes place over s steps, the firm adjusts $\hat{b}$ as

$$\hat{b}_{t+1} = \begin{cases} \hat{b}_t + (b_{obs} - \hat{b}_t)\frac{t}{s}, & t < s \\ b_{obs}, & t \geq s. \end{cases} \tag{17}$$

The firm's initial expectation of b is modeled as

$$\hat{b} = b(1 + \epsilon), \tag{18}$$

and with $-.9 < \epsilon < .9$, some of the firm's expected demand curves sit above the actual household demand, overestimating elasticity and price and for a given quantity of sugar. Conversely, some expectations sit below the actual household demand curve, underestimating elasticity and price.

Entities, State Variables and Scale

The model entities are a single firm, a single household, and a market through which the firm and household interact. The firm parameters, defined in the Firm class are:

`gamma`: The labour exponent for sugar production.

`error_max`: The maximum error in the firm's understanding of the household's demand function $\hat{b}$ pararmeter.

`n_steps`: The number of steps it takes for the firm to learn the household demand function's b.

`a_hat`: The firm's expectation of the household demand function a parameter.

`b_hat`: The firm's expectation of the household demand function b parameter.

`firm_money`: The balance of the firm endowment.

`p`: The price of the firm's sugar.

`L_opt`: The optimal labour the firm would like to meet the 0 profit condition.

`S_planned`: How much sugar the firm produces with optimal labour.

`S_supply`: How much sugar the firm produces with available labour.

`L_demand`: The lesser of optimal labour and affordable labour.

`revenue`: The amount of sales revenue from previous time step.

`firm_budget`: The amount the firm can spend on labour.

The household also has various state variables, defined in the Household class, which describe the household income-leisure preferences, the demand for sugar and budget and endowment values:

`L_max`: the maximum hours the household will work in a timestep

`alpha`: the Cobb-Douglas exponent for leisure, and therefore less than 1.

`beta`: .the Cobb-Douglas exponent for sugar consumption, 1 - alpha.

`a`: the coefficient of the household demand function for sugar.

`b`: the exponent of the household demand function for sugar, related to price elasticity of demand.

`house_money`: The balance of the household endowment.

`S_wanted`: The sugar the household desires based on the price and its demand function.

`L_supply`: The amount of labour the household provides based on its preference function.

`S_demand`: the amount of sugar the household demands on the market, the lesser of S_wanted and the amount that is affordable from wage income and endowment.

`house_budget`: the amount the household can spend on sugar.

The market is the entity that coordinates the purchase of both labour and sugar, and has only two state variables:

`L_market`: the lesser of L_supply from the household and L_demand from the firm.

`S_market`: the lesser of S_supply from the firm and S_demand from the household.

The model includes several model control variables defined within the Simulation class:

`verbose`: the model will output values of state variables at each time step.

`duration`: the number of time steps the model will run.

`use_money`: true or false, indicating if the model runs with or without firm and/or household money.

Process Overview and Scheduling

At the start of each simulation, economic equilibrium values are calculated from the current model parameters. The firm then calculates the optimal labour and sugar price given its understanding of the household demand for sugar. Given this price, the firm determines the amount of labour it will supply. The market determines the lesser of labour supply and demand. The firm produces sugar according to market labour and the household determines how much sugar to purchase based on its demand and budget. The market again determines the lesser of sugar supply and sugar demand, and the firm computes its revenues from the sale of sugar and the cost of labour. The firm and household then update their ledgers according to the market transactions and the firm improves its understanding of household demand. These processes are illustrated in Figure 1.

Inputs and Outputs

The model reads simulation parameter values from the `series_params.py` and sends simulation results to a designated `.csv` file. Input parameters are described in Table 1. For each time step and each unique set of parameters, the model saves this information for output: `L_demand, L_market, L_max, L_opt, L_supply, S_demand, S_market, S_planned, S_supply, S_wanted, a_hat, alpha, b, b_hat, beta, error_max, firm_budget, firm_money, firm_money_0, gamma, house_budget, house_money, house_money_0, n_steps, p, t, use_money` and `run`.

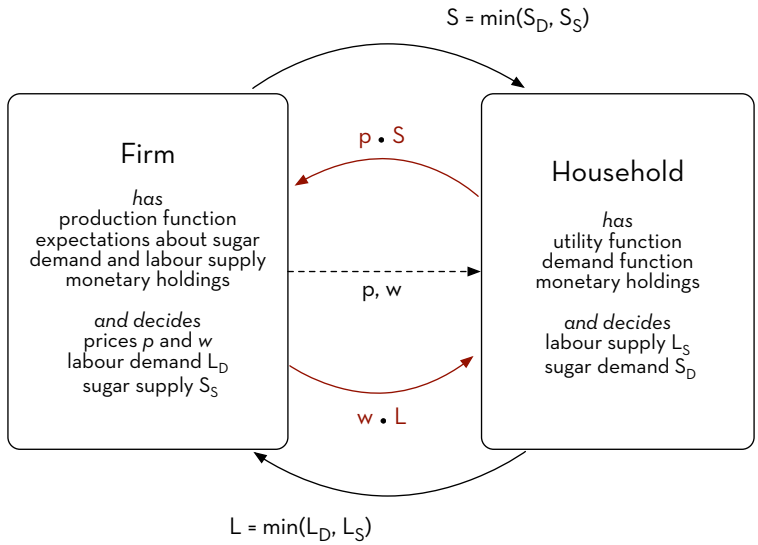

Figure 1: Equilibrium Selection Model overview.

Parameter	Description
directory	name of output file directory
verbose	true or false, whether the model prints variables each step
duration	the number of time steps the model will run
use_money	whether or not the model is run with firm and/or household money
gamma	the firms labour exponent for sugar production
spacing	the spacing of firm errors, used to generate a list of errors
error_max_list	list of error values, either generated from spacing or entered manually
n_steps	the number of steps over with the firm learns the household demand
revenue_0	initial revenue to start off the simulation
firm_money_0_list	a list specifying the values for firm endowments
L_max	maximum number of labour hours per time step%
b	the household elasticity of demand for sugar, -1.%
alpha	the preference exponent for leisure%
beta	the preference exponent for sugar%
house_money_0_list	a list specifying the values for firm endowments

Table 1: Input parameters in the `series_params.py` file.

Submodels

Determine Classical Equilibrium Values

L_star = beta / (alpha + beta) * L_max

p_star = L_star ˆ (1 - gamma)

S_star = L_star / p_star

For each time step,

Firm decides Labour Demand and Price

Firm decides L_opt from the zero-profit condition L_demand = minimum of L_opt and the firm's budget Firm plans sugar production, S_planned, using L_demand Firm decides price from p = (S_planned / a) ^ (1 / b_hat)

Household Decides Labour Supply

Household perceives p advertised by firm and determines S_wanted from S = a * p ^ b Household determines L_supply as the minimum of L_star and p * S_wanted.

Market Determines Quantity of Labour Exchanged

L_market = minimum of L_supply and L_demand

Firm Produces Sugar

S_supply = L_market ^ gamma

Household Determines Sugar Demand

S_demand = minimum of S_wanted and the household budget

Market Determines Quantity of Sugar Exchanged

S_market = minimum of S_supply and S_demand.

Firm Earns Revenue

revenue = p * S_market - L_market

Update Ledgers

if use_money:
firm_money = firm_money + p * S_market - L_market
house_money = house_money + L_market - p * S_market)

Firm Updates Expectation

if t < n_steps:
Firm performs a learning step to improve b_hat