Overview, Design concepts, and Details (ODD) of the Multi Asset Variable Network Model

This document describes fundamental aspects of the Multi Asset Variable Network Model following the ODD protocol developed by Grimm et al. (2006). Figure 1 below provides the graphical user interface (GUI) which captures the primary elements of the model. The user specified input parameters are divided into three groups; Market, Agent or Network Characteristics, in the upper left quadrant. The bottom half of the GUI has various graphs and output displays which the user can utilize to track the model's progress throughout a run. The model view is partial excluded as it has little utility. User notes are also provided in the bottom left hand corner. The author's model was implemented in NetLogo 5.3 (Wilensky, 1999).

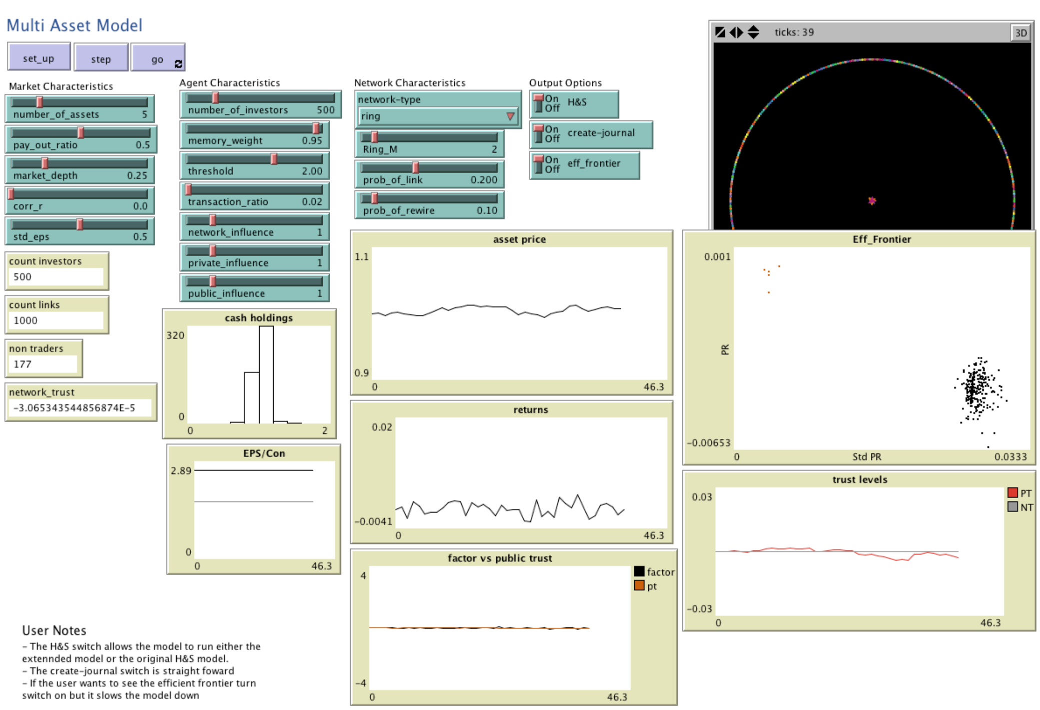

Figure 1 Screen Dump of the GUI

1 OVERVIEW

1.1 Purpose

Financial markets commonly produce periods of extreme volatility. In an attempt to understand this behavior, the use of a complex systems framework has become increasingly popular. A further benefit of utilizing a complex systems framework is that it allows networks to be included, with their relevance to financial markets being their ability to explain investor trading decisions and portfolio performance. The use of agent-based models (ABMs) has been a primary tool is trying to understand the dynamics of a complex system and a large volume of work utilizing ABMs to create artificial stock markets has been developed (see LeBaron (2006) and Sornette (2014) for comprehensive reviews of the literature) The key rationale for the use of ABMs being that they are not constrained to equilibrium conditions Sornette (2014). However, the utilization of network structures within these models has been limited. Utilizing the Ising based artificial agent-based model (ABM) of Harras and Sornette (2011) (H&S hereafter) as a foundation, various extensions were made to address a wide range of research of questions. The model implements an artificial stock market model that allows users to:

- Differ the network topology that investors are connected by and the number of investors;
- Have investors consider 1 to 10 risky assets;
- Change the source of public information;
- Alter the level of influence of the various information sources as per the H&S paper;
- Vary the dividend payout ratio of the risky asset; and
- Vary the correlation of the public information between the risky assets;

This flexibility allows a diverse range of topics to be research with the intent of producing insights that management, investors and regulators should consider. The model is implemented in NetLogo 5.3 (Wilensky, 1999).

1.2 State variables and scales

The basic premise of the original H&S model and the extended model is that boundedly rational investors[1] (for completeness, an investor(s) is defined as an agent who invests in the model) have access to three sources of information; the expected actions of their neighbors ($E_{ij}[a_{ik}(t)]$), public information ($pi_i(t)$) and private information $\epsilon_{ij}(t)$. Where the following definitions are relevant:

- *i* refers to the i^{th} asset;
- *j* refers to the actions of the j^{th} agent;
- *k* refers to the k^{th} neighbor of agent *j* and K is the total number of neighbors for agent *j*;
- *(t)* is the information relevant for the current time period (noting *(t-1)* is the information relevant from the prior period);
- ($E_{ij}[a_{ik}(t)]$) is the estimated action of agent *j's* k^{th} neighbor in relation to asset *i* at time period *t;*
- $pi_i(t)$ being the public information for the i^{th} asset at time period *t;* and
- $\epsilon_{ij}(t)$ is defined as the private information of the j^{th} agent for the i^{th} asset at time period.

The investors utilize these sources of information to determine their propensity to invest (ω_{ij}) in a risky asset(s) - (ω_{ij} being the result for j^{th} agent in relation to the i^{th} asset). Equation 1 details the exact calculation that the investors perform at each time step (defined as a tick).

[1] In this instance they are only considering past information.

The model then allows the investors to transact, with the new price endogenously determined for the asset(s) along with a variety of accompanying asset and portfolio statistics.

From Equation 1 it can be seen that the level of influence of each information source is weighted by two variables[2], with one of these being fixed and the other variable. The fixed values are given by c_{1ij}, c_{2ij} and c_{3ij}, with Table 2 providing a full description of the variable. In summary the *c* variable relates to the information source (neighbors (c_1), public (c_2) and private (c_3)) and *i* and *j* relating to the value for the j^{th} agent for the i^{th} asset.

Readers should note that the values for c_{1ij}, c_{2ij} and c_{3ij} are the same for each asset. The variable coefficients are network trust (nt_{jk}) and public trust (pt_i), with nt_{jk} being the network trust of agent j in their neighbor k and pt_i being the trust in public information for asset *i*. The lack of a *j* variable is because as detailed by Section 3.3.6, the level of public trust is homogenous across the population as all investors receive and assess the public information in the same manner. It is by altering the c_{1ij}, c_{2ij} and c_{3ij} coefficients, different dynamics were generated in the H&S model. In particular, when the upper limit for c_{1ij} is set at 4, bubbles in the risky asset's price appear.

Equation 1: The decision equation

$$\boldsymbol{\omega_{ij} = c_{1ij}\left(\sum_{k=1}^{K} nt_{jk}\,(t-1)E_{ij}[a_{ik}(t)]\right) + \; c_{2ij}pt_i(t-1)pi_i(t) + c_{3ij}\epsilon_{ij}(t)}$$

Or in simpler terms;

$$Decision\ score\ = Network\ score\ + Public\ score + Private\ score$$

[2] The exception is private information, which has a single variable.

The extended model has two low level entities, investors and risky assets. These are detailed in Section 1.2.1 and 1.2.2. An overview of the market process is provided in Section 1.2.3.

1.2.1 Assets

To address multiple assets, the implemented model has the capability to have between 1 and 10 risky assets (denoted as *I* assets with *i* being the i^{th} asset). With the introduction of multiple assets, assets effectively become agents and have the ability to be assigned, maintain and evolve heterogeneous characteristics. Table 1 details the key attributes of the asset class and the role they play in the model.

In a key difference from the H&S model, the extended model sees the asset(s) maintain an earnings per share (EPS) value for each period. EPS reflects the income generating ability of the asset and is a key component in determining the fundamental value of an asset. The model also makes use of the past values EPS values to generate a future earnings forecast for investors. The importance of including an earnings forecast is articulated by Sornette (2014) when he suggests “in a given financial bubble, it is expectation of future earnings rather than present economic reality that the average investor”.

In a further difference to the H&S model, by combining the EPS of the asset with its payout ratio, each asset returns a dividend per share (DPS), as shown in Equation 2. The justification of introducing dividends comes from the fact that dividends are a key component in the total returns for most assets. For example for the S&P 500, dividends are responsible for 42% of total returns (Ro, 2013). Each investor’s dividend is held in their dividend bank and investors do not have access to those funds for the purpose of further investing.

Equation 2: Dividend per share (DPS) per period

$$d_i(t) = eps_i(t) * pay\_out\_ratio_i.$$

Despite the role dividends play in supporting returns, it should be noted that the reason why firms pay a dividend is an area of ongoing discussion, with no decisive evidence supporting the argument they are used to signal favorable information to the market or to mitigate agency problems (Li & Lie, 2006). An alternative view has been provided by firstly (Baker & Wurgler, 2004), and supported by (Li & Lie, 2006), with the proposed dividend catering theory. Under the theory, investors' demand, and therefore any premium that dividend paying firms attract, is dependent on investors' appetite for dividends at any particular time, which in turn varies based on market conditions.

Table 1: Asset characteristics

Characteristic (model variable)	Function and Description
Price in $'s (price)	The price of each asset is initiated at $1 as per the H&S approach and is updated endogenously at each tick as per the process outlined in Section 3.3.5. In turn, a return series is determined given the changing prices.
EPS in cents (EPS)	To allow for an alternate process by which investors asses public information, each asset has an (EPS) value. The initial EPS is also the mean parameter for the probability density function (PDF) required for Step 4 of Figure 2 (nb. only for the extended model). The role that an asset's earnings have on the behavior of each investor is outlined in Section 3.3.2.
Asset Returns (ave_return and stddev_a_rt)	Through the market clearing process an asset's returns are generated. These returns are stored in a list and this allows the average and standard deviation of those returns to be calculated. The values of those calculations are stored in these variables.
Correlation (corr_r)	To allow for the varying effects of multiple assets, the public information of the assets are correlated to the first asset (Asset 0) by this variable. Greater detail of its use is provided in Section 3.3.2. The value is set by the `corr_r` parameter and is constant for all risky assets and ranges between 0 and 1(the value of β in Equation 3).
Pay_out_ratio	The payout ratio determines the percentage of earnings that are returned to the investors. In reality, a low growth stock will generally have a higher payout ratio and a high growth stock a lower one. Given the H&S model did not include a dividend (or EPS), any benchmarking sets this

	value at 0. The variable will be utilized to assess the impact of an asset's payout ratio on price volatility utilizing the extended model.
EPS_dev **H&S_dev**	Regardless of which model is implemented, Step 4 of Figure 2 requires a standard deviation for the PDF. For the H&S model the first asset has a standard deviation (H&S_dev) of 1 and a mean of 0. For each subsequent asset added the standard deviation increases by 0.1. For the revised model, the EPS's standard deviation (EPS_dev) for the first asset is set by the `std_eps` parameter, which is a fraction (between 0 and 1) of the mean EPS value. It then increases by .1 for each incremental asset.
Consensus EPS	The extended model also requires each asset to maintain a consensus EPS forecast (EPSF). The forecast is homogenous for the population and its use is detailed in Section 3.3.2. The forecast is formed as the ensemble average of past EPS result as per the following: $$\langle epsf_i(t)\rangle = \alpha * \langle epsf_i(t-1)\rangle + (1-\alpha) * eps_i(t-1)$$ α relates to the `memory_weight` parameter. Given the EPS results are drawn from a normal distribution, the consensus forecast should mean revert over time but the use of the above equation does allow for the development of trends.
Consensus_Accuracy	The revised model replaces the public information process of the H&S model with investors assessing the accuracy of the consensus forecast. This attribute captures the outcome of the processes described in greater detail in Section 3.3.2.

1.2.2 Investors

The population size of investors is variable and is set by the user. At all times they hold a combination of the risk free asset (a proxy for cash) - which can be redeemed to purchase the risky asset(s), and the risky asset(s). The objective of the investors is to improve/increase their wealth by buying the risky asset(s) when they think the value will increase and selling it (them) if they think the value will decrease. At initiation the investors are provided one unit of the risk free asset and one unit of each of the risky assets. Table 2 summarizes the key variables that the investors own and how they are utilized in the model.

Table 2: Investor variables

Symbol	Name	Purpose
c_{1ij}	Network influence	Each investor is initiated with a fixed value (float) that is drawn randomly from a uniform distribution between 0 and a value up to 5 that the user decides. The variable is used to weight the information the investor generates from their network. As investors have a different value for c_{1ij} this introduces a level of heterogeneity within the population (but not across assets). While beyond the scope of this paper it is a worthier consideration that if the value of c_{1ij} were allowed to be less than 0, it would introduce contrarian investors. Analyzing the impact of different levels for this variable and c_{2ij} forms a key component of this thesis and the H&S paper. An acceptable interpretation of these variables is that a higher value (such as 4), indicates a higher initial bias to that information source.
c_{2ij}	Public information influence	Similar to the above with the exception of weighting the public information by a value between 0 and the value of the `public_influence` parameter.
c_{3ij}	Private information influence	Similar to the above, with the exception of weighting the private information being set by the user defined `private_influence` parameter. Note that the private information has no adaption variable.
nt_{jk}	Network information trust	While this variable is initiated at 0, investors update the value at each tick (see Section 3.3.6) to reflect an increasing or decreasing level of trust in the information coming from each of the investors in their neighborhood. c_{1ij} is then used to compound the information from the investor's network. It should be noted that an investor places trust in a neighbor's overall ability to make the right selection (their average ability) and they do not keep track of a neighbor's ability to pick individual assets.
pt_i	Public information trust	Similar to the above with the exception of being the trust an agent has in public information for each specific asset ($asset_i$). As detailed by Section 3.3.6, the level of public trust is homogenous across the population as all investors receive and assess the public information in the same manner. However, in contrast to the trust an investor has in their neighbors, an investor maintains public trust at an asset level. The alternative approach - taking the average from the assets - was assessed but it was felt that by aggregating the trust a level of freedom was lost in model. It is not unrealistic to expect investors to have varying levels of faith in a stock's ability to surprise based on past results.
$\bar{\omega}_j$	Transaction threshold	Each investor is initiated with a fixed value (float) that is drawn randomly from a uniform distribution between 0 and the value of the `threshold` parameter. The default level for the model is 2.

		The variable is used as the value by which an investor decides to either buy, hold or sell (see Table 4 and Section 3.3.4 for a detailed description). As investors have different values for $\bar{\boldsymbol{\omega}}_j$, another level of heterogeneity exists within the population. H&S attribute this value to the risk aversion of the investor. Indeed an investor with a high $\bar{\boldsymbol{\omega}}_j$ requires significant evidence before they commit to a transaction, while a low value will see the agent act on the slightest change in information.
tr	Transaction ratio	The value is set by the `transaction_ratio` parameter and represents the fixed fraction that an agent is willing to trade. The default value is 0.02. While the transaction ratio is fixed, future research may look to have this variable vary based on how confident an investor is.

1.2.3 The Market

To remain consistent with the H&S model, a market marker model is employed for this model. The market remains closed with regards to the number of investors and their ability to access additional cash or raise debt to acquire risky assets, which is again consistent with the H&S model. In addition, to the market being closed, the dividends ($d_i(t)$) which are declared at each step, are not available to be re-invested into the market. This dampens the absolute movement of the price and should see the price mean revert to 1 in the absence of any other dynamics within the model. While dividends are not available for re-investment, their value is tracked because the value is needed for the portfolio statistics for the investors.

1.3 Process overview and scheduling

Figure 2 provides a summary of how the model progresses. The simulation is measured in discrete time, with each tick corresponding to a month. The actual time period is relevant only to calibrate the initialization price of the asset as outlined in Section 3.1. A summary of the model is that it is initiated with the conditions set by the user (see Sections 3.1 and 3.2) and then runs for until the user sees fit. Within the repeated steps the process can be grouped into the following;

- the updating of the various information sources;
- agents assessing that information and making their decision;
- the agents transact, which is followed by the assets assessing the results of their decisions; and
- book keeping where the agents whether is updated.

- **Initial Steps**
- Step 1: Assets initialised (Section 3.1)
- Step 2: Network formed (Section 3.1)
- Step 3: Network populated with investors (Section 3.1)

- **Repeated Steps**
- Step 4: Asset information updated (Section 3.31)
- Step 5: Agents assess information sources (Section 3.3.4)
- Step 6: Agents make investment decision (Section 3.3.4)
- Step 7: Order book formed (Section 3.3.5)
- Step 8: Market cleared and asset returns calculated (Section 3.3.5)
- Step 9: Agents reassess the value of public information (Section 3.3.6)
- Step 10: Agents reassess the value of their neighbors (Section 3.3.6)
- Step 11: Agent portfolios updated (Section 3.3.7)
- Step 12: Portfolio valuation performed (Section 3.3.7)

Figure 2: Overview of the model's implementation

In terms of the updating of the agents were initialized as per the NetLogo default of a random asynchronous order. This present the issue that some agents will be making their decision before other agents have assessed the new information. The author felt this position is justified and follows the rationale of H&S who indicated that this is acceptable because it reflected the different decision making timeframes that investors maintain.

2 DESIGN CONCEPTS

The primary design concept for the model is one of adaption and evolution, which is achieved via investors continually reassessing and adjusting the trust in each of their information sources (Step 9 and 10 of Figure 2) based on the ability of each source to predict the appropriate action. An appropriate action being when the information tells the investor to buy and the price subsequently increases (and vice versa for a sell signal).

In terms of other identifiable design concepts in the model, they are:

- **Interaction:** Agent interact by polling the expectations of their neighbors and entering their buy/sell orders into the market.
- **Emergence:** The model is capable of producing results that see asset bubble appear and then subsequently collapse. These dramatic price changes are inconsistent with the normally distributed news sources. Without the presence of this level of emergence the price series would also follow a Gaussian distribution.
- **Stochasticity:** Stochasticity is present at both the initialization of the model and throughout the running of the model. The stochasticity at initialization is seen in both the agents and the network structure (greater detail is provided in Sections 3.1and 3.2). In terms of the stochasticity during the run this comes in the form of the generation of the public and private information (see Section 3.3.1 and 3.3.2 respectively).
- **Observation**: An inherent value of ABMs is being able to collect data at both the agent and system level, therefore allowing researchers to capture emergent outcomes (Bonabeau, 2002). At the system level the model generates a new price at each step of the

model as described in Section 3.3.5. Associated with this new price are the returns of the asset plus the mean and the standard deviation of those returns. The model also collects the average network and public trust of the investors and the cumulative actions of the investors, which in turn allows for the analysis of the order book at each step.

In terms of the data collection at the agent level, the focus of this model and the subsequent research was to collect the average return and the standard deviation of the returns of the investors. This was necessary to create the quasi-efficient frontier. Given the infrastructure of the model the potential exists to extract far more information at the agent level. Obvious candidates include the centrality of the individual agents and their investment thresholds. The extraction of these variables will be helpful to uncover the determinant of what makes an investor successful or not.

The Netlogo network extension package is also utilized to calculate betweenness and closeness centrality measures for whichever network topology is implemented. The results are maintained at the agent level, which again allows for agent and system analysis to be undertaken.

3 DETAILS

3.1 Initialization

As seen in Figure 2, initialization is a three step process; initialize the risky asset(s), generate the network and then populate it with the agents. For the assets the user must firstly decide how many risky risk to have. Then the they can decide the correlation factor in the public news, the payout ratio and the standard deviation of the EPS of the risky asset. Table 1 details these variables in greater detail.

In terms of generating the networks, with the exception of the Erdos Renyi network, where the network extension of NetLogo was utilized, the following user determined algorithms were used. For the lattice network the first step in generating the network is for the user to set the number of neighbors they want each investor to have via the `Ring_M` parameter. Given undirected links are being formed by the investors, the parameter is set as half the number of required neighbors because an investor becomes a neighbor with another investor regardless of whether they create the link or the neighbor creates the link with them. Next the investors are placed into a list, which is sorted by the investor's numerical identification number. Each investor is then asked to form an undirected link with the next highest investor in the list. The process is repeated for the investor by value of the `Ring_M` parameter, with the following link being formed with the next highest neighbor.

The small world (see Watts and Stogatz (1998) for theory behind and relevance of small world newtorks) procedure firstly implements the lattice network procedure using the same `Ring_M` parameter. Next, each investor forms a list of their links, which they cycle through asking each link to rewire, with the probability provided by the `prob_of_rewire` parameter, to another investor within the population that they are not connected with. Rewiring involves creating an undirected link to the new investor and then removing the link to the existing neighbor.

Scale free networks (Barabási & Albert, 1999) are a special case of random networks (Erdös & Rényi, 1960) where the degree distribution of the agents follows a power law. To create a heavily skewed degree distribution for the scale free network the user needs to decide on the number of hubs (set by the `Ring_M` parameter) they require and the probability (`prob_of_link` parameter) of an investor connecting to each of those hubs. The procedure operates by each investor identifying the hubs by finding and then forming a list of the investors with the most number of neighbors. The number of investors in the list is determined by the

`Ring_M` parameter. Next, each investor with a probability determined by the `prob_of_link` parameter, links with each of the investors in the list. This process is sufficient to create a scale free network via a preferential attachment process.

The links between neighbors are undirected and not specifically weighted nor are the links dynamic. However, given investors vary the level of trust they have in each neighbor, the network does become quasi dynamic i.e. as the trust in a neighbor increases, the weight of a directed link between the two effectively increases. Future iterations of this model could look to have investors jettison untrustworthy neighbors and search for investors that have superior performance. Also, directed links may be a worthwhile investigation - just because you listen to a neighbor there is no guarantee they listen to you.

The final step is to initialize the agents. The user firstly decides the number of agents to be created. Then the memory length, threshold, transaction ratios along with the initial weight to the various information sources is set. The threshold and weights to the information sources are not homogenous for the agents as explained in Table 2. Table 2 also provides a more detailed explanation of the variables and the justification of their settings.

3.2 Input

With the initial price per asset being $1, an acceptable P/E ratio for an equity being 15 and the model simulating quarterly updates, the initial EPS value (in cents) is given as (1/4)/15 * 100. The network structures utilized in the model and their the range characteristics for the formation of each network is detailed in Table 3.

Table 3: Network characteristics

Network	Key Characteristics
Lattice	The `Ring_M` parameter
Small world	Number of initial links per investor The probability of rewiring (`prob_of_rewire` parameter)
Random network / Erdos Renyi	Probability of connection (`prob_of_link` parameter)
Scale free	Number of hubs (set by the `Ring_M` parameter) Probability of connection (`prob_of_link` parameter)

3.3 Sub-models

3.3.1 Private Information

At each time step investors update their private information/opinion with regard to each asset. Each opinion ($\epsilon_{ij}(t)$) is drawn randomly from a normally distributed population (N(0,1)) as part of Step 5 in Figure 2. This process ensures that the information is uncorrelated across and between assets and investors – a requirement to ensure that any emergent outcome occurs through the interaction of the agents rather than any correlation in the information provided to the agents.

3.3.2 Public Information

Regardless of which model is used, the updating of the public information occurs at Step 5 in Figure 2. To allow for the implications resulting from multiple assets, certain changes to the H&S model were required. In the H&S model, public information is determined in a similar manner to the private information, i.e. a Gaussian white noise process with a mean of 0 and variance of 1 (N(0,1))[3]. With multiple assets, a key question is how correlated are those assets to each other? Therefore, the model needed to allow for the possibility that the public information of the assets is fully, partially, or not at all correlated.

The process for updating the public information for each asset is given by Equation 3. The process works by the news for the first asset (asset 0) being randomly chosen from the Gaussian distribution and if there are multiple assets, the process is repeated for each of the assets. The final value is then determined by weighting the values by β or $1 - \beta$, β is determined by the `corr_r` parameter. While the equation only addresses the correlation between the first asset and each asset, it can be seen that if $\beta = 1$ then the process is the equivalent to the original H&S model because all risky assets will have the same public information.

Equation 3: Public information update

$$\boldsymbol{pi_i(t)} = \beta * AssetNews\_0(t) + (1 - \beta)AssetNews_i(t)$$

As discussion in Section 1.2.1, the introduction of EPS in the extended model required further changes. In effect, $\boldsymbol{eps_i(t)}$ for the asset replaces 'news' in Equation 3 as the earnings for the asset for each period are determined in a similar manner to the original model, with the exception that the earnings are drawn from a PDF based on the parameters as outlined in Table 1, that is, one with a non-zero mean. This factor also sees the consensus forecast always being strictly greater than 0. However, in an important difference under the extended implementation, investors assess the actual EPS ($eps_i(t)$) delivered at each step against the consensus forecast, as

[3] The exception is in the multiple situation where the standard deviation increments by .1 per asset.

per Equation 4. In terms of the consensus forecast ($epsf_i(t)$), each agent holds the same forecast, which is determined as the ensemble average of past EPS results as provided by Equation 5.

Equation 4: Public information

$$pi_{ti} = \frac{eps_i(t) - epsf_i(t)}{epsf_i(t)}$$

Equation 5: Consensus forecast

$$\langle epsf_i(t) \rangle = \alpha * \langle epsf_i(t-1) \rangle + (1-\alpha) * eps_i(t-1)$$

If the actual earnings for an asset exceed the consensus estimate, this is considered an earnings surprise and this would be reflected positively with a score greater than zero. Alternatively, if earnings miss to the downside this is a negative and agents will read it as a signal to sell down their holding in the asset. If earnings meet expectations, then the information adds no value in the decision-making process because the investor assumes the information is already reflected in the price (they are assuming that the EMH actually holds true!) and they will hold their current position. As investors have the same public information the level of trust they have in it for each asset is the same.

The primary support for the above deviation comes from the existing volume of work that has analyzed the impact of earnings announcements (see Kothari (2001) for an extensive review of the literature). In summary, the findings show that stock prices react positively to positive earnings news, yet it takes time for this information to be fully reflected in the price of the asset (Kothari, Lewellen, & Warner, 2006). Further support comes from Barberis, Shleifer and Vishny (1998), who produced a model of investor sentiment that was successful in explaining and replicating an asset's price movement following an earnings surprise.

3.3.3 Network Information

To capture the information from their network, each investor polls their neighbors in terms of their intentions (buy, hold or sell) for each asset in Step 5 of Figure 2. The results of the process are captured by the $E_{ij}[a_{ik}(t)]$ term in Equation 1, with the $a_{ik}(t)$ reflecting the action of the neighbor *k* in relation to asset *I* at time *t*, as per Table 4. The investor will then weigh the action by the amount of trust they have in the particular neighbor $nt_{jk}(t-1)$. The investor then sums the results from each neighbor before finally multiplying the value by their fixed c_{1ij} term. Given the updating process of the model, an investor may poll some neighbors before those neighbors have processed their new private and network information. This is not considered a problem because agent initiation in the model is consistent with the process described by H&S, thereby accepting the justification of varying reaction times between investors.

3.3.4 Investment Decision

After investors assess their new information and before they update their level of trust, they must make a decision at each time step (Step 6 in Figure 2). At initiation investors are provided with a threshold value $\bar{\boldsymbol{\omega}}_{j,}$ and it is this value that they compare with their score ($\omega_{ij}(t)$) when deciding their actions ($a_{ij}(t)$) for each asset at time *t*. Table 4 details the conditions by which they make their decisions.

Table 4: Agent decision thresholds

Scenario	Action	Variable	Trading Volume
$\boldsymbol{\omega_{ij}(t) > \bar{\omega}_j}$	Buy	$a_{ij}(t) = +1$	$v_{ij}(t) = tr * \frac{rf_j(t)}{p_i(t-1)}$
$\boldsymbol{\omega_{ij}(t) < \bar{\omega}_j * -1}$	Sell	$a_{ij}(t) = -1$	$v_{ij}(t) = tr * holding_{ij}(t)$
Otherwise	Hold	$a_{ji}(t) = 0$	

Having decided to buy, hold or sell, the investors must decide how much they are willing to buy or sell (Step 6 in Figure 2). Table 4 again provides the formula by which they determine the transaction value ($v_{ij}(t)$). In determining the value of each transaction, the agents apply the coefficient *tr.* H&S indicate that as long as this coefficient does not approach 1 the general findings of the model are not significantly affected[4]. With regard to the calculation, investors face the following constraints within the model:

- No leverage - Agents must have a positive holding of the risk free asset ($rf_j(t)$) to enable them to trade; and
- No short selling – Agents must have a positive holding of the asset ($holding_{ij}(t)$) they wish to sell.

These constraints raise the possibility that an investor may wish to undertake an action but are unable to. It is for this reason that it is the intention of an investor's neighbor (given by $a_{ij}(t)$) that is polled rather than the transaction value.

It should be noted that when deciding how much to invest ($v_{ij}(t) > 0$), investors do not attempt to forecast what the price will be at the completion of the trade (p(t)), rather they use the existing price, which is provided by p(t-1). H&S again suggest that the alternative approach does not impact the results.

3.3.5 Market Clearing Process

Following the accumulation of the investors' orders for each asset, the market is cleared and the returns for the asset(s) are determined via Equation 6 (see Steps 7 and 8 in Figure 2). The first term of Equation 6 remains consistent with the H&S model, where the λ term is used to weight

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[4] This was tested with regards the author's model and a similar conclusion was reached.

the investor's actions by the market depth. A detailed analysis and justification for the use of this term and the general market pricing mechanism is provided by Farmer (2002).

The $\sum_{j=1}^{N_j} a_{ij}(t) * v_{ij}(t)$ term, which is the accumulation of the investor's individual orders, provides the surplus/deficit demand for the asset at each step. If there is net buying (i.e. a surplus in buy orders) then the return for the period will be positive and the price will increase, the opposite occurring if there is a net selling (deficit in demand). Excess prices movements for any period will be generated by large surplus/deficits and indicates that the population has become a herd. A bubble and crash will occur if the population remains in that herd for an extended period, that is, there are multiple periods of net buying or selling.

Equation 6: Return determinant for asset i

$$r_i(t) = \frac{1}{\lambda * J} \sum_{j=1}^{N_j} a_{ij}(t) * v_{ij}(t) + \log\left((d_i(t) + p_i(t))/\, p_i(t)\right)$$

In a deviation from the approach of H&S, an asset's dividend $(d_i(t))$ is included when an asset's return is determined as per the last component of Equation 6. Readers should revert to Section 1.2.1 for a detail the rationale and process by which the dividend is determined for each asset at each step.

The value of $r_i(t)$ is placed into a list so that the average and standard deviation of the asset's returns can be tracked and used to plot the quasi-efficient frontier. However, before this occurs, the inverse log is taken to ensure the values are in the same scale as the investor's returns as determined in Section 3.3.7.

Within step 8 the price of each asset is updated as per Equation 7 and then the model takes the inverse of the $\log[p_i(t)]$ term to finalize all book keeping for the investors (Section

3.3.7). Technically, an asset's price should fall by its dividend value once the dividend is paid. However, in the interest of preserving the intention of Equation 7, the price is not impacted by the dividend of the stock.

Equation 7: Log price update

$$\log[p_i(t)] = \log[\,p_i(t-1)] + \frac{1}{\lambda * J}\sum_{j=1}^{N_j} a_{ij}\,(t) * \; v_{ij}(t)$$

3.3.6 Trust Updating

After the investors become aware of their returns, they reassess the level of trust they have in the information provided by their network and public sources, as used in Equation 1, and update (nt_{jk}) and (pt_j) respectively (Step 9 and 10 in Figure 2). The rationale for these variables, which are initiated with a value of 0, is that the investor will place greater trust in a source if it provides the correct advice. That is, if the agent receives a buy (sell) signal from the information source, and the price subsequently increases (decreases), then the weight (trust) increases. The weight decreases if the signal is erroneous. This process follows the H&S paper due to the argument put forward in support of the process, i.e. investors are "looking for persistent sources of information, which impact on returns". There is an important difference in updating the two trust variables. For public trust, the population maintains trust at an individual asset level to preserve the heterogeneous nature of the asset's performance. In contrast, for an investor's trust in their neighbor, they will assess the individual recommendations of their neighbors before updating the trust based on the average performance of the recommendations.

Investors follow the process as illustrated in Figure 3 when assessing the usefulness of the public information $(pi_i(t-1))$ or network information $(\sum_{i=1}^{I} E_{ij}[a_{ik}(t)])$.

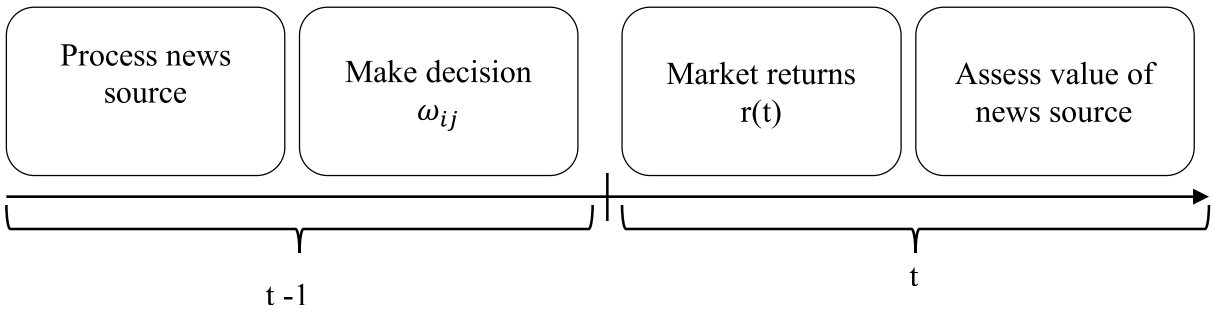

Figure 3: The simplistic flow of agents assessing the usefulness of information

The key implication of the process, as detailed by H&S is “that for any information source to have any predictive power it must have some persistence”. For this statement to hold, investors process their information sources and make their decision in t-1. The market will subsequently move in t-1 (see Figure 3) based on the combined decisions of the investors in t-1. However, it is not until period t that investors assess the value of the information that they processed in t-1 and update their trusts levels according to the standard autoregressive update as detailed in Equation 8 and Equation 9.

Equation 8: Public trust update

$$pt_i(t) = \alpha pt_i(t-1) + (1-\alpha)pi_i\,(t-1) * \frac{r_i(t)}{\sigma_{ir}(t)}$$

Equation 9: Network trust update

$$nt_{jk}(t) = \frac{\sum_{i=1}^{I} \alpha nt_{jk}(t) + (1-\alpha)E_{ij}[a_{ik}(t)] * \frac{r_i(t)}{\sigma_{ir}(t)}}{I}$$

The first part of the above equations discounts the previous trust value variable by the variable (α), which is set by the `memory_weight` parameter. The significance of the α variable, which is also used in the consensus forecasting process, is that it sets the time scale over which past performance impacts a variable's value. The time scale as per H&S is given as $\frac{1}{|\ln(\alpha)|}$. The second part of the equation adds the assessment of the immediately preceding information, which has been discounted by (1- α) after it is multiplied by the $\frac{r_i(t)}{\sigma_{ir(t)}}$ term. The $\frac{r_i(t)}{\sigma_{ir(t)}}$ term is used to normalize the past return of an asset by the standard deviation of its past returns. The rationale, as articulated by H&S, is that a larger return scaled by its volatility ($\sigma_{ir(t)}$) will enhance the trust to a greater degree. This process provides the potential mechanism for the development of a bubble or crash, as investors place more and more trust in an information source when it provides greater profits (or saves losses), hence the potential to create a positive feedback loop.

Equation 10 details how $\sigma_{ir}(t)^2$ is computed as a moving standard deviation with an exponentially decreasing kernel. The $\langle r_i(t)\rangle$ *and* $\langle r_i(t-1)\rangle$ terms represent the ensemble average of the return series, where the ensemble average is defined as the expected object of the stochastic process.

Equation 10: The variance and ensemble average for an asset's return

$$\sigma_{ir}(t)^2 = \alpha * \sigma_{ir}(t-1)^2 + (1-\alpha) * (r_i(t) - \langle r_i(t)\rangle)^2$$

Where $\langle r_i(t)\rangle$ is given by:

$$\langle r_i(t)\rangle = \alpha * \langle r_i(t-1)\rangle + (1-\alpha) * r_i(t)$$

3.3.7 Portfolio Updating

The final step of the process is to update the investors' portfolio and calculate their portfolio statistics (Steps 11 and 12 in Figure 2). It should be noted that the dividend payments are not added to $rf_j(t)$, the balance of the risk free asset, as the investors cannot reinvest dividends. The implication being that the model remains a closed system and therefore does not deviate too far from the original H&S model. The alternate approach would introduce a wealth effect, adding a layer of complexity, thus making it harder to interpret the implications of introducing the other new factors to the market.

Equation 11 provides the book keeping formulas used to update the portfolio. The investor's risk free asset (rf_j) is updated by the proceeds of any sales and the cost of any purchases, noting that the proceeds are impacted by the price realized in the period. In a similar manner, the holding for each of the investor's risky assets are updated.

Equation 11: Portfolio updates

$$rf_j(t) = rf_j(t-1) - \sum_{i=1}^{I} s_{ij}(t) * v_{ij}(t) * p_j(t)$$

$$holding_{ij}(t) = holding_{ij}(t-1) + s_{ij}(t) * v_{ij}t$$

Given the model's design, each investor's average portfolio return and its standard deviation are calculated at each tick. The results of these calculations allow for the construction of the quasi-efficient frontier, as illustrated in the bottom The quasi-efficient frontier plots the average return for each investor's portfolio against its standard deviation, which is the proxy for risk. Equation 12 provides the formula for the value of an investor's portfolio value. The inclusion of the dividend bank should be noted as it does form part of the investor's returns.

Equation 12: Portfolio value

$$Portfolio\,Value_{ij}(t) = rf_j(t) + \sum_{i=1}^{I} holding_{ij}(t) + dividendbank_{ij}$$

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