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The Agent-Based Model of the Closed Market(similar to Stock Market) with One Commodity and with careful and risky mechanisms of individual behavior of participants of market

The model of market of one commodity , in which there are in each moment of time the same quantity and the same quantity of money was formulated and researched in this text. Each partner of the market in the one moment of time can be in one of three status: to be buyer, be seller and do not take part in trade in this moment of time. In addition each of them can change his status in the next moment of time. Partners of market change their statuses and prices, by using the personal information of each of them about trade in the previous moment of time only.. Some characteristics of dynamics of average price of market in the case of careful choice only or in the case of one variant of risky choice only were received as a result of our research by computer model. The nature of dynamics of the set of prices of participants was investigated analytically. The main result is the convergence of trajectory of our system to stationary set of states with average price of trade which is close to some constant when behavior of all agent is careful and bounded hesitation of this trajectory when there are risky agents only. These facts are established by series of experiments with computer realization of the model. The behavior of trajectory of system was investigated in the case when all agents are identical simple determinate automata with linear tactic with careful and risky actions..

We could formulate the agent-based model of the closed market with one commodity i.e. model of market, in which there are the same quantity of commodity and the same quantity of money in each moment of time . Participant of the market can be seller, buyer or not take part in trade in each moment of time. But in next moment of time each participant can change his status: i.e. a seller can become the buyer or the partner which is waiting. There is the same situation for a buyer and for a waiting partner. Participant of trade change their status and declare new prices by using of his own information only . In each moment of time the buyer which has a money and agrees to pay the maximal price is trading with the seller which has a commodity and agrees to receive the minimal price . Moreover the prognosis of average price in the next moment sometimes stimulates sellers to change their status , i.e. to become buyers in the next moment or to refuse to take part in trade for several moments of time (to be waiting). The same take place for a buyer. The simple algorithm of changing by participant his status(seller, buyer, waiting) in the next moment of time was formulated in the papers.

. Closure of the market means that in each moment of time the sum of money which all participants have is equal unit and sum of quantities of commodity which all participants have is equal unit also. The time is proposed discrete: t=0,1,2,…. Participants of market are numbered by i:(i=1,2,3,...,.) the each participant of market has one of three statuses that means that in each moment of time there is the number $\alpha_i(t)$, which can be 1,-1,0 . Each agent is able to have only one from three statuses i.e. $\alpha_i(t) = 1$ indicates that the agent is the seller in this moment, $\alpha_i(t)=-1$ indicates that the agent is the buyer and $\alpha_i(t)= 0$ indicates that this agent takes no part in the trade in this moment of time(he waits the corresponding situation to take part in trade) . Each participant of the model can to have commodity and money simultaneously. It is difference of this model from our model of nonclosed market which we have investigated many years ago. Let denote by x_i(t) the quantity of commodity which agent i has and denote by y_i(t) the quantity of money which agent i has in moment of time t. The price v_i(t) also is characteristic of state of agent i in moment t. When this agent is a seller ($\alpha_i(t) = 1$) he shall not agree in this moment of time t to sell his commodity by price which less that his price $v_i(t)$. When this agent is a buyer ($\alpha_i(t) = 1$) he shall not agree in this moment of time t to buy the commodity by price which more that his price $v_i(t)$. When $\alpha_i(t) = 0$ (agent is waiting) the price of this agent has meaning of the indicator for choice of one from three possible decision of this agent in moment t: will become seller, will become buyer or to remain the waiting. When the participant of market is a waiting agent he can change his price depending on the relation of his price and average price of market in moment of time t which we denote by u(t). Moreover agent has one additional simple variable $k_i(t)(k_i(t) = +1, -1)$ corresponds to agent i in moment of time t. If $k_i(t) = 1$ then agent i changes his price more carefully than in the case $k_i(t) = -1$. So the state of our model of market in moment of time t is described by the 5N variables . We shall suppose that each moment of time consists from two steps (tacts of time). During the first step takes place the following. Each seller (participant for which $\alpha_i(t) = 1$) proposes to all buyers to buy all his commodities. Just the same way each buyer is ready to spend for purchase of the commodity all his money.

The exchange consists from bargains and consequences of these bargains are defined by relations of prices of sellers and prices of buyers. Let $\alpha_i(t) = 1$ at $i = i_1, i_2, i_3, \dots, i_k$. and let $v_{i_1} \le v_{i_2} \le v_{i_3} \le \cdots \le v_{i_k}$ and also $\alpha_i(t) = -1$ at only $j = j_1, j_2, j_3, \dots, j_l$ and let $v_{j_1} \ge v_{j_2} \ge v_{j_3} \ge \cdots \ge v_{j_l}$, So the first bargain happens between seller with minimal price and buyer with minimal price, the price of their trade equals half of sum of two these prices: $(v_{i_1} + v_{j_1})/2$.

Let for definiteness the buyer with number j_1 used up the all his money for buy of the part of quantity of commodity which seller i_1 had, but some part of commodity remains at seller i_1 after bargain . Then the seller i_1 offer his remainder of commodity to buyer with number j_2 and bargain between them happens in just the same way as was described above. The bargain between them will be fulfilled by price $(v_{i_1} + v_{j_2})/2$.

In the contrary case when after first bargain the seller i_1 sold all his commodity but the some money remained at buyer j_1 after first bargain then buyer j_1 ask the commodity from seller i_2 and bargain between them will be fulfiled by price $(v_{i_2}+v_{j_1})/2$. Further it will the next bargain depending on result of this second bargain. The next bargain will be between seller i_1 and buyer j_3 or between seller i_2 and buyer j_2 (similarly between buyer j_1 and seller i_2 , or between buyer j_2 and seller i_2). Such process of sequential bargains will be continued as long as the at least will be fulfilled one from following three conditions. The first: all sellers have no commodity . The second: all buyers have no money. The third: the price of seller who still has a commodity is more than price of buyer who still has a money. We have not considered until now the case when several sellers have in given moment of time the same prices and also the case when several buyers have in given moment of time the same prices moreover both cases can happen simultaneously. In these cases the exchange the commodity on the money take place between one generalized seller with given price of a selling and one generalized buyer with given price of a purchase. After bargain the all money which the generalized seller has received (if he had sold all his commodity) or all commodity which had been bought by generalized buyer (if he had spent all his moneys) distribute between all sellers with given price or between all buyers with given price. . We use in this investigation the following principle of distribution. The sellers with the same price which have small quantity of commodity sold all their commodity other sellers with this price sold only part of their quantity of commodity. The same relate to buyers. That is reason of the result that those sellers and buyers will have the different prices in the next moment of time.

In the case of this principle of distribution the sellers which ask the same price are divided on two groups: sellers which sold all his commodity and sellers which sold only part of his commodity. Distribution of expenditures among buyers with the same price happens analogously. Quantity of commodity and quantity of money of agent i which took part in exchange vary once end of the first tact (step of time) of moment of time t is reached and at $\alpha_i(t) = 1$will be x_i(t+1)≤ $x_i(t), y_i$(t+1)≥ y_i(t) and at $\alpha_i(t) = -1$ will be $x_i(t+1) \geq x_i(t), y_i$(t+1)≤ $y_i(t)$. If agent i is a seller and he not took part in trade because his price is rather high for buyers, then quantities of his commodity and of his money not change. If agent i is a buyer and he took not part in trade because his price is rather low for sellers, then quantities of his commodity and of his money not change also. If agent is a waiting agent then it is obviously that quantities of his commodity and of his money not change . We can define the average price of exchanges $w_i(t)$in which this seller or this buyer took part in the first tact(step of time) of moment of time t:

w_i(t)=(y_i(t+1)-y_i(t))/(x_i(t)-x_i(t+1)) if $\alpha_i(t) = 1$

w_i(t)=(y_i(t)-y_i(t+1))/(x_i(t+1)-x_i(t))   if $\alpha_i(t) = -1$

It is possible to define general variables of exchange in this tact and if any exchange happens then we can define average price of trade of whole system at this moment of time.

$\Delta Y(t) = \sum_{i=1}^{i=N}(y_i(t) - y_i\,(t)\alpha_i$(t)($\alpha_i(t) - 1)/2$

ΔX(t)=$\sum_{i=1}^{i=n}(x_i\,(t) - x_i$(t+1)$a_i$(t)($\alpha_i(t) + 1)/2$

if turn out $\Delta X(t) > 0$to be then we set that:

$$u(t) = \frac{\Delta Y(t)}{\Delta X(t)}$$

Where we shall name u(t) the average price of market in the moment of time t..

We shall suppose that center (operator) one for all market exists. All bargains at market register by this center/ He also calculates the average price and declares this average price to all participants of market. The value of average price in moment of time t is a single external information for participant of market. Other information of participant is the his own information(with index i).

In the course of the second tact of moment of time t each participant of market changes his status and after it he determines his new prices. He do it by using the result of trade on the first tact of this moment of time. We are constrained to propose for the simplification of model that participant can use for establishment of new status and new price only information about result of his trade on the first tact of moment t and the average price in moment t. Moreover to make the algorithm of choice of statuses and prices maximal simple we suppose that participants are hoping for that the average price in moment t+1 will be the same that average price in moment t.

So we have described the changing of prices of participants of our model of market. We can write this algorithm by formulas. We shall use the Heavisid function:

$$\theta(x) = \begin{cases} o\ if\ x < 0 \\ 1\ if\ \ x \geq 0 \end{cases}$$

The definition of the status of a participant i in moment t+1 is a following.

if $\alpha_i(t) = 1$ then

when $x_i(t+1) = 0\quad \alpha_i(t+1) = -[(1 - \theta\big(u(t) - w_i(t)\big)]$

when $x_i(t+1) > 0\quad \alpha_i(t+1) = 1 - \theta(v_i(t) - u(t)(1 + \theta(-y_i(t+1)))$

if $\alpha_i(t) = -1$ then

when $y_i(t+1) = 0\quad \alpha_i(t+1) = 1 - \theta(w_i(t) - u(t))$

when $y_i(t+1) > 0\quad \alpha_i(t+1) = -1 + \theta(u(t) - v_i(t)(1 + \theta(-x_i(t+1))$

if $\alpha_i(t) = 0$ then

when $y_i(t+1) = 0\quad \alpha_i(t+1) = \theta(u(t) - v_i(t))$

when $y_i(t+1) > 0 \quad \alpha_i(t+1) = -\theta(v_i(t) - u(t))$

The definition of the price of a participant i in moment t+1 (which fulfill to these conditions)is a following.

If α_i(t)=1 and x_i(t+1)=0 then v_i(t+1)=$w_i(t)$ when $k_i(t) = 1$ but $v_i(t+1) = w_i$(t)-d$\theta(w_i(t) - d - u(t))$ when $k_i(t) = -1$.

If α_i(t)=-1 and y_i(t+1)=0 then v_i(t+1)=$w_i(t)$ when $k_i(t) = 1$ but $v_i(t+1) =$ w_i(t)+d$\theta(u(t) - w_i(t) - d)$ when $k_i(t)$+-1

If α_i(t)=1 and x_i(t+1)>0 then v_i(t+1)=$v_i(t) - d[1 - \theta(u(t) - v_i(t))]$ *when* $k_i(t) = 1$ but $v_i(t+1) = v_i(t) + d\{(1 - \theta(v_i(t) - u))\theta(u - v_i$(t)-d)+ $\theta(v_i(t) - u)[-\theta\left(-y_i(t+1)\right)\theta(u - v_i(t) - d) + 1 - \theta(-y_i$(t+1)]\} when $k_i(t) = -1$.

If α_i(t)=-1 and y_i(t+1)>0 then v_i(t+1)=$v_i(t) + d\{1 - \theta(v_i(t) - u(t)]$ *when* $k_i(t) = 1$ but $v_i(t+1) = v_i(t) + d\{(1 - \theta(u(t) - v_i$(t)))$\theta(v_i$(t)-u(t)-d)+$\theta\big(u(t) - v_i(t)\big)[\theta(-x_i(t+1))\ \theta(v_i$(t)-u(t)-d)+1-$\theta(-x_i$(t+1))]\} when $k_i(t) = -1$.

If α_i(t)=0 and x_i(t+1)>0 then v_i(t+1)=$v_i(t) - d[1 - \theta\big(u(t) - v_i(t)\big)]$ *when* $k_i(t) = 1$ but $v_i(t+1) = v_i$(t)+d$\{\theta\big(u(t) - v_i(t)\big) - (1 - \theta\big(u(t) - v_i(t)\big))\theta(u(t-1) - u(t)\}$ when $k_i(t) = -1$.

If α_i(t)=0 and x_i(t+1)=0 then v_i(t+1)=$v_i(t) + d[1 - \theta(v_i(t) - u(t))]$ but $v_i(t+1) = v_i(t) - d[\theta\big(u(t) - v_i(t)\big)\theta(v_i(t) - u(t) - d) - (1 - \theta\big(u(t) - v_i(t)\big)\theta\big(u(t) - u(t-1)\big)]$

These formulas give to us possibility to find r(t+1) as a function of r(t) ,u(t) and u(t-1).

Denote by ρ(t)=$max_{\alpha_i(t)=1} v_i(t) - min_{\alpha_i(t)=-1} v_i(t)$ and we shall name this value by width of spectrum of prices of participants of trade in moment of time t.. Denote also σ(t)=$min_{\alpha_i(t)=-1} - max_{\alpha_i t)=1} v_i(t)$ and we shall name this value by divergence of spectrum of prices of participants of trade in moment of time t

Assertion 1. The exist some moment of time T_0 that begginning with moment of time T_0 wil be ρ(t)>0, σ(t) >0 for t> T_0.If $\alpha_i(t) = 0$ then for t>T_o will be either x(t)=0 or y(t)=0.

Two assertion(2,3) which were proved say about convergence of set of price to the rather simple configuration. Assertion 4 says that u(t) decreases (but not monotonically when u(t) is large and increases when u(t) is small.

The next assertion is not proved but received by computer model.

If $k_i(t) = 1 (i = 1,2, \ldots, N)$ during all time then there exists such moment of time τ that for each t> τ takes place r(t)∈ $M_0(d)$, where for all points of set $M_0(d)M$ will be $u_o - 5d \le u(t) \le$ u_o+5d.

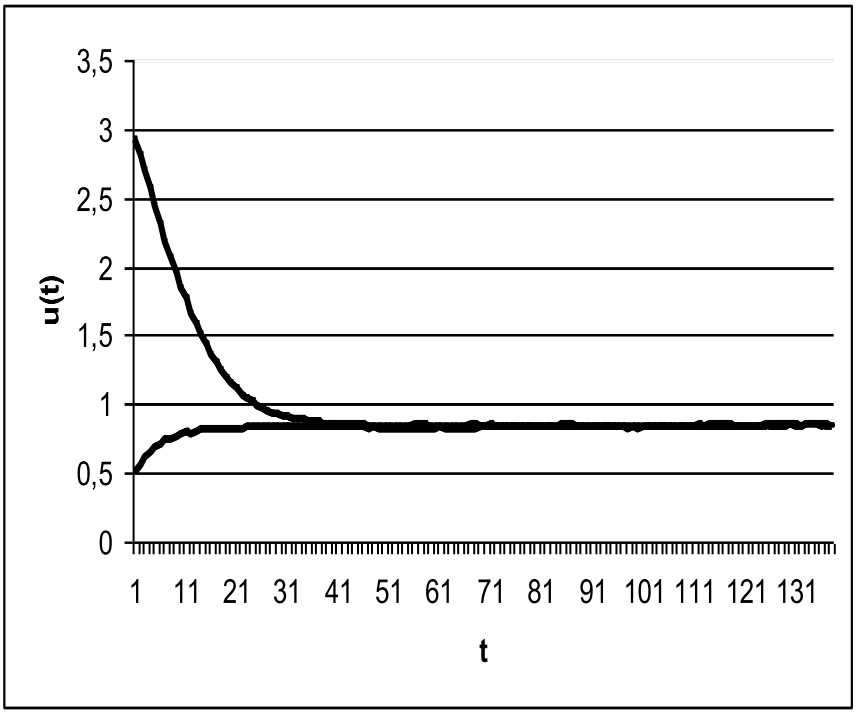

Figure 1 The graph of change of average prices of closed market during the time for two initial conditions (N=300,d=0.005,u(0)= 3 u(0)=0.5, one step in axis of time on the graph equals 100 moment of time).

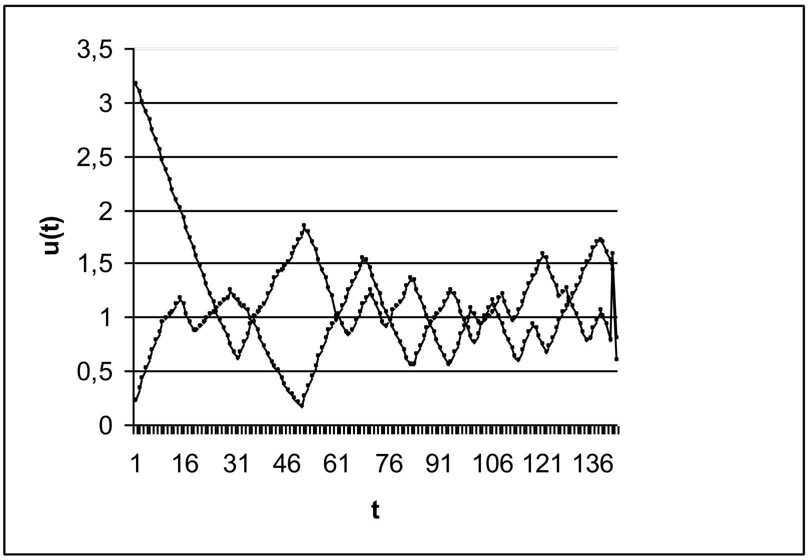

Figure 5. Two trajectories of average price of market when all participants are risky agents.(N=500, d=0.005,u(0)=0.2 and u(0)=3.3. 0<t<14000).

Literature

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