Supplementary Information (SP) for the NetLogo model
"The coevolution of the firm and the product attribute space"
César García-Díaz, Gábor Péli, Arjen van Witteloostuijn

Introduction

This model inspects the performance of firms as the product attribute space changes, which evolves as a consequence of firms' actions. Firms may create new product variants by dragging demand from other existing variants. Firms decide whether to open new product variants, to invade existing ones, or to keep their variant portfolio. At each variant there is a *Cournot* competition each round. Competition is nested since many firms compete at many variants simultaneously, affecting firm composition at each location (variant).

After the *Cournot* outcomes, at each round firms decide whether to (i) keep their existing product variant niche, (ii) invade an existing variant, (iii) create a new variant, or (iv) abandon a variant. Firms' profits across their niche take into consideration the niche-width cost (*N*) and the cost of opening a new variant (*K*).

The model has a constant population of an *F* number of firms with two different types: those with scale economies (α-type) and those without such economies (β-type). Firms are distributed over the two types with equal probability, and they cannot change their type during the simulation. The terms $n_{\alpha,j,t}$ and $n_{\beta,j,t}$ denote, respectively, the number of firms per type that offer product variant *j* at time *t*. For the sake of analytical convenience, we abandon time index *t* from the formulae. β-type firms have a constant unit production cost $c_{\beta,j}$ at variant *j*; α-type firms are able to reduce unit production costs as production volume increases by exploiting scale economies. An α-type firm's $c_{\alpha,j}$ realized unit production cost at variant *j* depends on its production level $q_{i,j}$ related to this variant:

$$c_{\alpha,j} = c_{\beta,j} - e_j q_{i,j} \ . \quad (1)$$

A product variant's maximal cost $c_{\beta,j}$ is drawn from a uniform probability distribution. The $e_j > 0$ value is *j*'s unit cost reduction impact per production unit. In order to avoid having negative $c_{\alpha,j}$ values, we establish a minimum unit cost c_o, so that $c_{\beta,j} - e_j M \geq c_o$, with M standing for total demand. From this formula follows that $c_{\beta,j} \geq c_o + e_j M$. We set $c_o = 0$.

Each firm *i* calculates its profit from variant *j* according to:

$$\pi_{i,j} = P_j q_i - c_{r,j} q_{i,j} \ , \tag{SP1}$$

with P_j being the price at variant *j*, $q_{i,j}$ the quantity firm *i* produces at *j*, q_i the total quantity offered at *j*, $c_{r,j}$ the unit production cost, and *r* the firm type (α or β). A population of n_j firms offers product variant *j*, of which $n_{\alpha,j}$ are *α-type* and $n_{\alpha,j}$ are *β-type*. The price equation of *j* is defined as:

$$P_j = a - b_j Q_j = a - b_j \sum_{i=1}^{n_j} q_{i,j} \ , \tag{SP2}$$

Instantiating Equation (SP2) in Equation (SP1), a given firm *i* that takes advantage of scale economies (i.e., *α-type*) maximizes its benefits at variant *j* when:

$$\frac{\partial \pi_{i,j}}{\partial q_{i,j}} = a - b_j \sum_{z=1,z\neq i}^{n_j} q_{z,j} - 2b_j q_{i,j} - c_{\beta,j} + 2e_j q_{i,j} = 0, \ \ i = 1,2, \dots n_{\alpha,j}, \tag{SP3}$$

and

$$\frac{\partial \pi_{i,j}}{\partial q_{i,j}} = a - b_j Q_j + q_{i,j}\left(2e_j - b_j\right) - c_{\beta,j} = 0, \ \ i = 1,2, \dots n_{\alpha,j}. \tag{SP4}$$

Summing up over all $n_{\alpha,j}$ firms, we get:

$$n_{\alpha,j} a - n_{\alpha,j} b_j Q_j + (2e_j - b_j) \sum_{i=1}^{n_{\alpha,j}} q_{i,j} - n_{\alpha,j} c_{\beta,j} = 0. \tag{SP5}$$

In an analogous way, for a *β-type* firm, we obtain:

$$\frac{\partial \pi_{i,j}}{\partial q_{i,j}} = a - b_j \sum_{z=1,z\neq i}^{n_j} q_{z,j} - 2b_j q_{i,j} - c_{\beta,j} = 0,$$

$$i = n_{\alpha,j} + 1, n_{\alpha,j} + 2, \dots, n_{\alpha,j} + n_{\beta,j} \ . \tag{SP6}$$

Thus, summing up over all $n_{\beta,j}$ firms, we have:

$$n_{\beta,j} a - n_{\beta,j} b_j Q_j - b_j \sum_{i=n_{\alpha,j}+1}^{n_{\alpha,j}+n_{\beta,j}} q_{i,j} - n_{\beta,j} c_{\beta,j} = 0. \tag{SP7}$$

Knowing that the total number of firms in variant j is $n_j = n_{\alpha,j} + n_{\beta,j}$, and that $Q_j = \sum_{i=1}^{n_{\alpha,j}} q_{i,j} + \sum_{i=n_{\alpha,j}+1}^{n_{\alpha,j}+n_{\beta,j}} q_{i,j}$, we proceed to multiply Equation (SP7) by $-(2e_j - b_j)/b_j$ and add it to Equation (SP5). Then we get:

$$\left[n_{\alpha,j} - [\frac{(2e_j-b_j)}{b_j}]n_{\beta,j}\right]a + \left[n_{\beta,j}(2e_j - b_j) - b_j n_{\alpha,j}\right]Q_j + (2e_j - b_j)Q_j - \left[n_{\alpha,j} - [\frac{(2e_j-b_j)}{b_j}]n_{\beta,j}\right]c_{\beta,j} = 0 \, , \tag{SP8}$$

which yields a total quantity at variant j of:

$$Q_j = \frac{n_{\alpha,j} - [\frac{(2e_j-b_j)}{b_j}]n_{\beta,j}}{b_j n_{\alpha,j} - (2e_j - b_j)(n_{\beta,j}+1)}(a - c_{\beta,j}) \; . \tag{SP9}$$

Going back to Equations (SP5) and (SP7), and defining $Q_{\alpha,j} = \sum_{i=1}^{n_{\alpha,j}} q_{i,j}$, $Q_{\beta,j} = \sum_{i=n_{\alpha,j}+1}^{n_{\alpha,j}+n_{\beta,j}} q_{i,j}$, we get:

$$Q_{\alpha,j} = \frac{n_{\alpha,j}(a - b_j Q_{\beta,j} - c_{\beta,j})}{b_j n_{\alpha,j} - (2e_j - b_j)} \, , \tag{SP10}$$

and

$$Q_{\beta,j} = \frac{n_{\beta,j}(a - b_j Q_{\alpha,j} - c_{\beta,j})}{b_j n_{\beta,j} + b_j} \, . \tag{SP11}$$

Solving by substitution, Equations (SP10) and (SP11) become:

$$Q_{\alpha,j} = \frac{n_{\alpha,j}}{b_j(n_{\alpha,j}+n_{\beta,j}+1) - 2e_j(n_{\beta,j}+1)}(a - \ c_{\beta,j} \ ) \, , \tag{SP12}$$

and

$$Q_{\beta,j} = \frac{[\frac{(b_j-2e_j)}{b_j}]n_{\beta,j}}{b_j(n_{\alpha,j}+n_{\beta,j}+1) - 2e_j(n_{\beta,j}+1)}(a - \ c_{\beta,j} \ ) \, . \tag{SP13}$$

Then, if firm i is *α-type*, its production level at variant j would be $q_{i,j} = Q_{\alpha,j}/n_{\alpha,j}$; if it is *β-type*, then its production level would be $q_{i,j} = Q_{\beta,j}/n_{\beta,j}$.

Variable / parameter definitions

All the variable and parameter definitions and values are presented next.

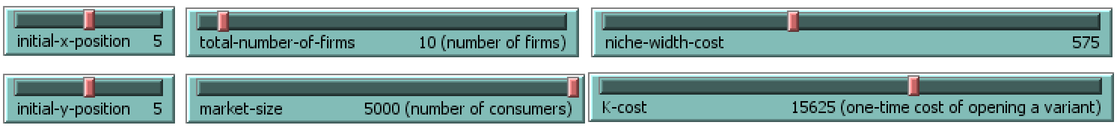
Denotation	Explanation	Range of values
M	Total number of consumer units	5000
F	Total number of firms	[10, 20]
c_0	Minimum attainable unit cost	0
$c_{\alpha,j,t}$	α-type firms' unit production cost	Variable
$c_{\beta,j}$	β-type firms' unit production cost	Uniformly distributed in range $[a, e_j M]$
e_j	Unit cost reduction impact per unit of production quantity at variant *j*	Uniformly distributed in range [0.45, 0.50)
$\pi_{i,j}$	Profit of firm *i* at variant *j*	Variable
P_j	Price of variant *j*	Variable
$q_{i,j,t}$	Firm *i*'s production quantity at variant *j* at time *t*	Variable
$q_{i,t}$	Total firm *i*'s production quantity time *t*	Variable
a	Price equation intercept	*M*
$b_{j,t}$	Price equation slope	(0,1]
n_j	Total number of firms at variant *j*	Variable
Q_j	Total quantity at variant *j*	Variable
$n_{\alpha,j,t}$	Number of α-type firms at variant *j* at time *t*	Variable
$n_{\beta,j,t}$	Number of β-type firms at variant *j* at time *t*	Variable
N	Fixed cost of attending a variant (niche spanning)	[500, 1500]
$H_{i,t}$	Firm *i*'s niche at time *t*	Variable
$q_{i,t}$	Firm *i*'s total production quantity at time *t*	Variable
$\tilde{\pi}'_{i,t+1}$	Firm *i*'s profit expectation at time *t*+1	Variable
$\tilde{\pi}_{i,j,t+1}$	Firm *i*'s profit expectation at variant *j* at time *t*+1	Variable
$s_{H_{i,t}}$	Largest block distance between any two variants in firm *i*'s niche at time *t*, $H_{i,t}$	Variable
$\tilde{\pi}''_{i,t+1}$	Firm *i*'s profit expectation at time *t*+1 after inclusion of a new variant	Variable
V_t	Number of active variants at time *t*	Variable
$\tilde{\pi}_p$	Firm *i*'s profit expected monopolistic profits at new variant	Variable

$\tilde{\pi}'''_{i,t+1}$	Firm *i*'s profit expectation at time *t*+1 after expanding to an existing variant	Variable
$\tilde{\pi}''''_{i,t+1}$	Firm *i*'s profit expectation at time *t*+1 after a variant has been dropped	Variable
d	Dropped or additional variant	Variable
K	One-time cost of opening up a new variant	[10000, 25000]

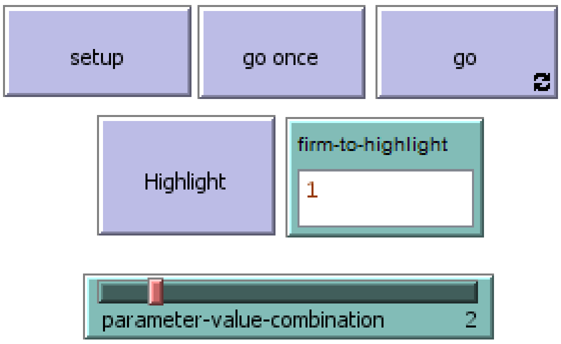
Table SP1. Variable / parameter definitions

Interacting with the model

The model has the following slider buttons, which define values of the main parameters (niche-width cost (*N*), number of firms (*F*), market size (*M*), and variant opening cost (*K*)):



The model can be first initiated and then run with the following buttons: "setup", "go once" (one "tick"), and "go" (continuous execution):



The button *Highlight* show on the screen the niche of a specific firm (e.g., firm 1). The slider button *parameter-value-combination* establishes which of the specific combinations of parameter values becomes active. Such combinations are derived from a Latin Hypercube Sampling procedure, according to the following table:

Scenario	**Total number of firms (*F*)**	**Niche-width cost (*N*)**	**Variant opening cost (*K*)**
1	17	525	21625
2	16	1125	17875
3	10	575	15625
4	11	775	23875
5	12	1325	20125
6	15	1275	20875
7	13	1025	14875
8	11	925	22375
9	16	625	14125
10	20	725	24625
11	18	1425	23125
12	19	825	12625
13	14	1475	13375
14	14	1375	16375
15	19	875	17125
16	17	1175	19375
17	18	975	11875
18	12	675	11125
19	15	1225	10375
20	13	1075	18625

Table SP2. Parameter value scenarios obtained by the Latin Hypercube Sampling procedure for the three variables of interest (number of firms, niche-width cost, and variant opening cost)

The model illustrates 12 plots with results. Among them, one plot shows the computation of the fraction dimensionality of the space (a non-integer value between 0 and 2). See the full paper for details.