General Housing Model with Banks

J M Applegate
February 25, 2020

Purpose

For many individuals in Western economies, owning property is the primary form of wealth generation. Property markets are also very sensitive to credit availability and banking terms, and as recently demonstrated bubbles in the real estate market can trigger global systemic crises. Here we describe a model of a basic housing market with bank lending, renters, owners and landlords. This model was developed for Arizona State University's 2020 Winter School: Agent-Based Modeling of Social-Ecological Systems. This model incorporates elements of several existing housing models [1, 2, 3, 4] and can serve as a starting point for exploring property-based wealth inequality, how banks lending behavior drives crashes [5], the emergence of an affordability curve, and how investment and construction decisions shape the housing stock. The model can also be calibrated to explore the dynamics of specific cities or regions.

The model is currently implemented in NetLogo 6.0.4, therefore containing an explicit local dimension, and is intentionally incomplete in terms of population evolution, investment market, construction decisions and competition between multiple banks to allow Winter School students to extend the model in a direction of interest.

Entities, State Variables and Scale

Model entities are households, houses, banks, and various records. Households currently are the only entities with varying behaviours, and represent individuals and families that seek housing, earn income, and consume goods and services. Households will also make investment decisions, though they don't in the current version. The household parameters are:

`residence`: house or nobody, indicating the residence of the household. If this is nobody the household is currently without a home.

`income`: $\Gamma(\alpha, \lambda)$, income of the household is drawn from the Γ distribution parameterized by α and λ. This income value doesn't change over the course of the simulation, though an evolving population of households would likely involve a gradual increase in this value over time.

`lender`: bank, the bank holding the mortgages, if any, for the household. In a multiple bank implementation, this would change if another bank offers better terms.

`capital`: $\mathcal{U}(0,$ annual income$)$, a household's savings which grows over time with any wage or rental income net housing and consumption costs.

`madeOffer`: house or nobody, the house a household would like to purchase.

`affordableRent`: affordability * income, amount a household can spend on rent.

`affordablePrice`: f(affordability, income, interest, LTV), price of house a household can afford.

`timeHomeless`: in months, length of time a household has been homeless.

`consumption`: $\sigma(a, b)$, percentage of income spend on non-housing consumption, currently derived empirically from United States data.

`affordability`: maximum percentage of income a household will spend on housing, which may increase if a household is without a home for a certain amount of time.

`accustomed`: quality of previous residence

Houses are entities with various state variables, including endogenous price, mortgage and rent parameters. The house parameters are:

`owner`: household or nobody, the owner of the house

`inhabitant`: household or nobody, the inhabitant of house

`quality`: .3 to 3, indicator of the quality of a house

`status`: one of "for sale", "for rent", "owner occupied" or "tenant occupied", the house's market status.

`askingPrice`: f(locality, mean price, aspiration, time on market), the asking price of a house for sale.

`askingRent`: f(locality, mean rent, aspiration, time on market), the asking rent of a house for rent.

`price`: initial price of house or last sale price.

`rent`: latest rent of house.

`buyer`: household or nobody, the latest buyer of the house

`seller`: household or nobody, the latest seller of the house

`rentalTerm`: 0 to standardRentalTerm, remaining time on rental contract.

`payment`: monthly payment on mortgage.

`mortgage`: amount of house mortgage.

`dateMarket`: date house went on market.

`offerFrom`: household or nobody, household that wants to buy the house.

`saleDate`: ticks, date house was last sold.

`rentalDate`: ticks, date house was last rented.

Banks are also entities, and with a multiple bank implementation will make endogenous parameter setting decisions regarding competitive interest and loan-to-value rates. Bank parameters are:

`minLTV`: requiredLTV, minimum loan to value ratio required by bank.

`term`: standardMortgageTerm, length of financing offered by bank.

`interestRate`: interbankRate + .02, interest rate charged by bank for a mortgage.

`ledger`: list of records, all sales for which the bank is the mortgage provider.

`profit`: the running profit of the bank

The model includes numerous global variables, most of which are set on the interface, and represent macroeconomic conditions either requiring empirical parameterization, or representing central bank or other policy constraints.

`meanIncome`: mean of the gamma distribution used for generating income for households.

`density`: percentage of universe populated with houses.

`locality`: houses within this range from a given house are considered to be local.

`ownershipRate`: percentage of houses that are owner-occupied.

`exitRate`: percentage of owner-occupied houses that go on the market each step.

`entryRate`: intended to represent new households entering the universe, not currently used.

`interbankRate`: interbank rate set, usually, by a central bank.

`requiredLTV`: loan to value ratio required for a sale, the maximum percentage of house price that can be financed.

`standardMortgageTerm`: standard length in years of financing.

`standardRentalTerm`: standard length of rental contract in months.

`affordabilityRate`: starting percentage of income a household spends on housing.

`maxAffordability`: maximum percentage of income a household can spend on housing.

`minConsumptionRate`: minimum percentage of income a household spends on non-housing consumption.

`aspiration`: the maximum percentage over base sale price that a seller adds to generate asking price.

`deltaRate`: the percentage of asking price or rent that is adjusted based on market conditions.

`timeMarketThreshold`: time on market before which prices and rents are adjusted upward, and after which are adjusted downward.

`maxHomeless`: length of time without housing until household adjusts affordability upwards.

Process Overview and Scheduling

At the start of each simulation, the banks are instantiated, then houses are created and randomly located on empty patches according to the specified density. Each house then generates a household with an income based on the parameterized Γ distribution, and house price, mortgage principle and payment, and quality are set based on the household income. Households are assigned a bank, and appropriate sale and bank records are generated. Houses are then rearranged such that houses of similar price are closer together. A subset of the houses are given the "owner occupied" designation and the owners become the inhabitants of the house. The remaining houses are designated "tenant occupied" with the original owners becoming the inhabitant, and then ownership is randomly reassigned to households with top incomes, and suitable rental records are generated.

At the start of each step all households receive wage and rental income and pay housing costs. Houses that are for sale are identified, and an asking price is determined based on local sales and aspirations, and adjusted according to demand. House seekers are identified, currently just those households seeking houses in which to reside. House seekers make an offer on the highest-quality available house they can afford.

Once all offers are made, sales proceed for houses with offers provided buyers have the capital to make the downpayment. Sellers move out of houses and buyers move into houses. Sellers who didn't purchase a new house are without a residence and will seek to rent. Original houses of buyers who did not sell are left empty.

The same process is repeated for the rental market. Houses that are for rent are identified, asking rents set according to recent local rentals, aspirations and market conditions, rental seeking households make offers on the highest-quality available rental property they can afford, rental houses with offers are rented and renting households move into those homes.

Households now consume goods and services based on their available income, and adjust their housing affordability rate based on the amount of time they are without a residence.

Sale or rental records are generated as appropriate for every transaction, and bank records are added to a lending bank's ledger for new mortgages. Figure 1 illustrates the model's logical flow.

The current version of the model provides several opportunities to make functional additions to the code, including:

1. population evolution dynamics simulating birth and mortality of households, including income growth and inheritance,

2. household decisions to buy or sell investment property to be included in the sales market, and

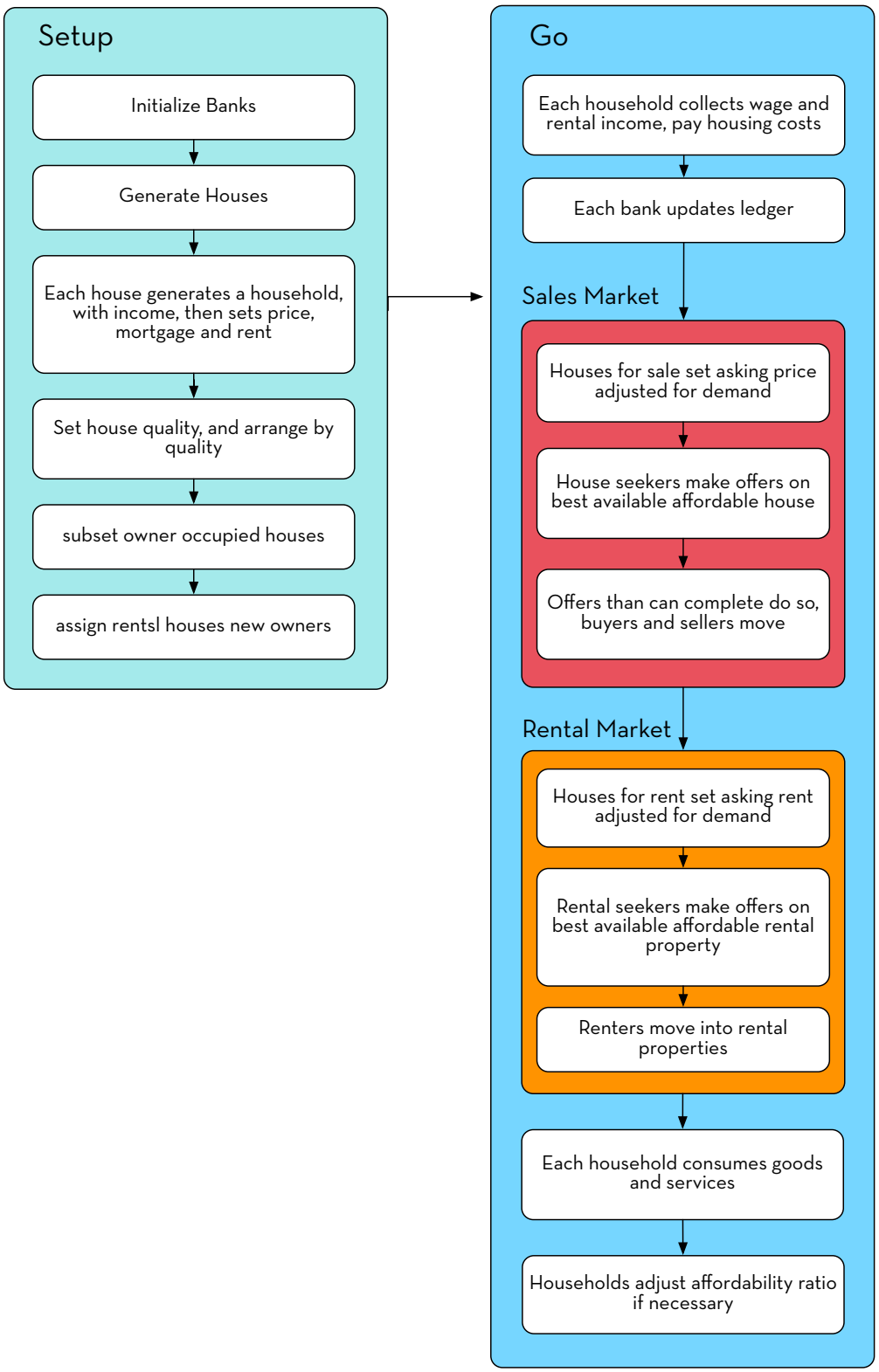

Figure 1: General Housing Model current version logical flow. Need elements of population and stock evolution each time step, as well as investment purchasing within sales market.

3. an evolution of the housing stock where existing houses are replaced with houses of a quality determined by demand.

4. an implementation of multiple banks would add a market for mortgage services and require banks to make decisions about competitive loan-to-value ratios or interest rates, and house buyers requiring a mortgage would choose between available options.

Initialization and Inputs

Values for many global variables are set on the interface.

VARIABLE	VALUE
meanIncome	$40,000
density	75%
locality	5
ownershipRate	65%
exitRate	10%
interbankRate	1%
requiredLTV	80%
standardMortgageTerm	20 years
standardRentalTerm	12 months
affordabilityRate	33%
maxAffordability	66%
minConsumptionRate	20%
aspiration	10%
deltaRate	5%
timeMarketThreshold	3 months
maxHomeless	6 months

Table 1: Common initial starting conditions set on interface panel.

Inputs and Outputs

The model does not require any inputs, and generates various outputs displayed on the interface panel. These outputs include static reporters showing the income distribution, number of households, number of owner occupied houses and number of rental houses. The interface panel also show a series of dynamic plots updated each time step, including house prices, rents, time on market for both sales and rental markets, number of property vacancies and number of households without a residence, as well as the changing affordability and consumption distributions by income.

Submodels

Create Households

Households are created to initially occupy each house, assigned an income from a Γ distribution, and house price, mortgage payment, rent, principal and quality. If using a spatial dimension, houses

will need to be grouped according to common metrics.

```
for each house
  draw income from Γ(α, λ) where lambda = α/meanIncome
  payment = income * affordabilityRate
  rent = payment * aspiration
  principle = payment / interest multiplier of bank
  price = principle / ltv of bank
  quality = f(income)
  create sales record for house
cluster houses according to price/quality
```

Selling Market

House sales will always complete if possible, requiring buyers to have available funds to complete transactions. This will result in some sales falling through due to insufficient funds, some buyers owning multiple houses, and some sellers becoming homeless.

Buyer cases:

1. Buyer is moving into purchased home from previous owner-occupied home.
   (a) The previous home sells.
   (b) The previous home does not sell.
2. Buyer is buying property for investment and will offer it for rent.*

Seller cases:

1. Seller is living in house that is sold.
   (a) The seller purchases and moves into another owner-occupied house.
   (b) The seller becomes homeless.
2. Seller does not live in house that is sold.

Pseudocode for this submodel is:

```
selling market
  determine houses for sale
  set asking price
    f(sale records in location)†
  price is adjusted up or down based on time on market conditions
  determine house buyers
    all current homeless
```

```
    individuals with house for sale‡
  buyers determine affordable price
    f(LTV, interest, capital, equity, income)
  buyers identify options and place bid on highest quality option
  sellers accept or reject bids
  buyers determine downpayment
    if capital + equity <= 2 * price
      downpayment = max((capital + equity) / 2, (1 - LTV) * price)‡
      deduct downpayment from capital and price
      set up mortgage and payment
  complete sales
  successful buyers and seller move
```

Rental Market

All households who do not have a residence, or households whose rental contracts have just expired, are seeking rentals. A household could rent its previous home. Pseudocode for this submodel is:

```
rental market
  determine houses for rent
  set asking rent
    f(rental records in location)†
  asking rent is adjusted up or down based on time on market conditions
  determine renters
    all current homeless
    individuals whose lease just expired
  renters determine affordable rent
    f(income)
  renters identify options and place bid on highest quality option
  landlords accept or reject bids
 complete rentals
```

Potential Submodels

This current version of the General Housing Model is intended for instruction, and here we outline several potential submodels, each of which could be written or integrated within a week-long course. These outlines are suggestions, as there are many possible algorithms that could address these functions.

Population Evolution

The model population could evolve in several dimensions. The households themselves could age, acquiring higher incomes over time and eventually die off, with young households being instantiated [2]. Thus the population could expand or collapse. Population evolution could also be modeled as households leaving and entering at specified rates [4]. The current model will have household savings grow to the point where mortgages are no longer requires, so alternative capital accumulation and dissipation processes could be explored. Income distributions and demographic changes can all be modeled using empirical data for specific cities or regions.

Investment Market

Houses with capital above a certain threshold could consider buying a house that would then be offered for rent. The investment purchasing decision would be based on expected rental income, and could be as simple as whether or not the expected income net mortgage payments are greater than some percentage, such as an expected market return on the capital. The selling decision could be similarly motivated, or driven by households that age out. Investment buyers and sellers could participate in the existing sales market, with the addition of flagging an investment property as "for rent" rather than "owner occupied", where it would be pickup up as a rental property in the rental market.

Evolution of Housing Stock

Houses age and fall into disrepair, or demand for a certain quality of housing may change over time, both of which would lead to some houses being destroyed and new houses built. Housing stock could also expand according to demand, but wouldn't necessarily contract. With the observer acting as construction company, houses of a specific age or quality could be removed and replaced with new houses based on demand for specific quality, determined by the local price.

Competing Banks

The model currently has the option on the interface to select the number of banks, which means that mortgage holders are randomly assigned one of the banks as a lender, but their behaviour is now identical. Banks could make decisions about LTV ratios or interest rates for various borrowers and properties based on profit optimization, and households could chose the best mortgage available from a variety of offers.

Works Cited

Axtell, Robert, et al. "An agent-based model of the housing market bubble in metropolitan washington, dc". *Whitepaper for Deutsche Bundesbank's Spring Conference on "Housing markets and the macroeconomy: Challenges for monetary policy and financial stability." May.* 2014.

Baptista, Rafa, et al. “Macroprudential policy in an agent-based model of the UK housing market” (2016).

Gilbert, Nigel, John C Hawksworth, and Paul A Swinney. “An Agent-Based Model of the English Housing Market.” *AAAI Spring Symposium: Technosocial Predictive Analytics*. 2009. 30–35.

Pangallo, Marco, Jean-Pierre Nadal, and Annick Vignes. “Residential income segregation: a behavioral model of the housing market”. *Journal of Economic Behavior & Organization* 159 (2019): 15–35.

Turner, Adair. *Between debt and the devil: Money, credit, and fixing global finance*. Princeton University Press, 2017.

Mathematical Formulations

Mortgage Principle and Interest Multiplier

P is principal, J is interest rate, N number of month ($term * 12$), M monthly mortgage payment, H current monthly interest, C amount of principle paid each month

$$H = P * J \tag{1}$$
$$C = M - P * J \tag{2}$$
$$Q_1 = P - (M - PJ) \tag{3}$$
$$= P_M + PJ \tag{4}$$
$$= P(1 + J) - M \tag{5}$$

Now set $P = Q$.

$$H = Q_1 J \tag{6}$$
$$H = (P(1 + J) - M)J \tag{7}$$
$$C = M - Q_1 J \tag{8}$$
$$C = M - (PJ(1 + J) - MJ) \tag{9}$$
$$Q_2 = P(1 + J) - M - [M - (PJ(1 + J) - MJ)] \tag{10}$$
$$= P(1 + J) - M - M + PJ(1 + J) - MJ \tag{11}$$
$$= P(1 + J)^2 - M(1 + J) - M \tag{12}$$

Again, set $P = Q$.

$$H = Q_2 J \tag{13}$$
$$C = M - Q_2 J \tag{14}$$
$$Q_3 = Q_2 + Q_2 J - M \tag{15}$$
$$= Q_2(1 + J) - M \tag{16}$$
$$= P(1 + J)^3 - M(1 + J)^2 - M(1 + J) - M \tag{17}$$

We recognize $M(1 + J)^2 + M(1 + J) + M$ as a geometric series with $a + M$ and $r = (1 + J)$.

$$a + ar + ar^2 + ar^3 \cdots = \sum_{k=0}^{n-1} ar^k = a\left(\frac{1ir^n}{1 - r}\right)$$

Derived by

$$s = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1} \tag{18}$$

$$rs = ar + ar^2 + ar^3 + ar^4 + \cdots + ar^n \tag{19}$$

$$s - rs = a\left(\frac{1 - r^n}{1 - r}\right) \tag{20}$$

At payoff

$$0 = P(1 + J)^N - M\left(\frac{1 - (1 + J)^N}{1 - (1 + J)}\right) \tag{21}$$

$$P(1 + J)^N = M\left(\frac{1 - (1 + J)^N}{1 - 1 - J)}\right) \tag{22}$$

$$M = P(1 + J)^N M\left(\frac{J)}{(1 + J)^N - 1}\right) \tag{23}$$

$$\therefore M = PJ\left(\frac{(1 + J^N)}{1 + J)^N - 1}\right) \tag{24}$$

The term

$$J\left(\frac{(1 + J^N)}{1 + J)^N - 1}\right) \tag{25}$$

is referred to as the *interest multiplier* in the model.