M.M.Voronovitsky

THE DYNAMIC AGENT BASED MODEL OF MARKET OF THE SINGLE COMMODITY WITH MANY PARTICIPANTS AND THE CHARACTER OF THE PROCESS OF SETTING OF PRICES

The dynamic agent based model of system which turn out the self-adjusting system, are considered in this text. As a distinguish from usual microeconomic approach we characterise the choice of participant not by utility function but by algorithm of behavior(automaton) . Each participant uses only part of the all information i.e. he uses only information which is his own information. There two kinds of participants - sellers and buyers, each seller interacts with several buyers (and vice versa) and these interactions are a pair interactions. Nevertheless the system (model of market) for any initial condition reaches the steady state and this steady state is a equilibrium of this market.The fact that rather complex and large dynamic system really reach the state which is nearly equilibrium state at condition that nobody has all information and each participant has only small own information is surprisingly, by our opinion .

There is one model of exchange between supplier and buyer which represent the price of exchange as $\frac{v+w}{2}$, where v is price of supplier and w is price of buyer. That model is usual in microeconomics theory, but the theory describes only case of two participants and seldom market with many sellers and buyers. Furthermore microeconomics theory not describes a unequilibrium states and process of reaching the equilibrium. In the paper a dynamic model of exchange is discussed where the participants are identical automata. A structure of the automaton,for which the system reaches an equilibrium, is found. According to theory of economic equilibrium the behaviour of the individual participant is supposed to be rational, i.e. his choice of decision is determined by his wish to maximize his utility function.

This is one of the causes of the situation, that the models of market, considered in this theory are statical, i.e. they don't reflect adaptation characteristics of behaviour of the participants and do not allow us to investigate the process of setting of the equilibrium.

Another path for such a research is given by us in the simple models of market of of one which was considered by us before. Their behaviour is not rational, but the algorithm of behaviour in different external market conditions is given.

In short, the participant is an automaton with a price (requested or offered) - the main variable. This way of research allows us to give the formal description of the participants interaction. For the built dynamic model there exists (as proved in previous works) the stationary equilibrium state that is globally steady(see [1]-[4]).

This allows us to suppose that this model (at least, qualitatively) describes the real process of setting of the equilibrium in the market. In

particular, our formal description allows us to analyze the dynamics of establishment of equilibrium at different mechanisms of agent

behavior and interaction of agents. We not suppose, that the dynamic model of the market, created by us, can be used for

microeconomic analysis, but this model gives to us possibility to explain character of market's processes . Establishment of equilibrium

can be investigated also by using of experiments and computer models. We have to describe the main elements of our market model.

First of all we will describe the mechanism of exchange. Let there are the seller, having x units of goods and ready to sell them at the

price, which is not lower than v, and buyer with u units of money, which is ready to pay the price,not higher,than w are meeting at the

given moment of time. This means, that, if the somebody proposes to seller to sell his goods at the price, which is lower than v, then he

refuses from the bargain. The same happens, if the price, which is higher than w will be proposed to buyer.

It is assumed, that buyer and seller discuss only the prices of the commodity. For simplifying the model, the volume of sales and

purchases is defined only by x and u and the price of exchange. The exchange takes place at the price, equal to $\frac{v+w}{2}$. The exchange is

possible, if the condition $v \leq w$ is being fulfiled. Let's assume that the result of the bargain is the following: either the seller will sell all of

his goods, or the buyer will spend all of his money, i.e. let $v \leq w$ and δ is the volume of goods sold. Then $\delta = x$ if $\frac{x(v+w)}{2} \leq u$ and $\delta = \frac{2u}{v+w}$

if $\frac{x(v+w)}{2} \geq u$:

Suppose, that we have m sellers of a goods and n buyers of this goods. v_i, x_i are minimal price and the volume of goods of seller i;

$(i = 1, 2, 3.., m), w_j, u_j$ are maximal price and amount of money of buyer $j; (j = 1, 2, 3..., n)$.

For definiteness, at the given moment of time:

$v_1 < v_2 < ... < v_m$

$w_1 < w_2 < ... < w_n$

Suppose, that each of the sellers knows maximal prices of all buyers i.e. $w_1, w_2, ..., w_n$, then it will be natural for him to offer his goods to

the buyer with the highest w (i. e. to try to sell it at the maximal price). Similarly, each of the buyers knows the minimal prices of all sellers

, i.e. $v_1, v_2, ..., v_m$ and he will request the goods from the seller with the smallest v (to buy the goods at the lowest price). Thus, each of

the sellers is choosing the buyer, who is ready to pay the maximal price, and the buyer tries to buy from the seller, which asks the

minimal price. We hope, that this order of exchange approximately describes of the logic of the real exchange.

If the wishes of the buyer and the seller coincide, and they still possess their goods or money, the exchange takes place in line with rules,

described above. If, after the exchange, the seller still has a part of his goods, then he offers it to the buyer at the price of the next lower

level (w), etc. The buyer requires the goods (if he still has any money) from the seller with the next higher level v, etc. If $v_1 \leq w_1$ then the

first act of exchange will take place between the seller with minimal v and the buyer with the maximal w. Thus, the process of exchange

at the given moment of time consists of several acts of mutual exchange in pairs, and it will finish in the following cases: Either for all of

the pairs"seller-buyer", which possessing the money and the goods, v will be higher, than w, or the sellers will have no commodity at all

or the buyers will have no money at all.For the formal description of the process of exchange we will mark the amount of goods, sold by

seller i to buyer j by $\delta_{i,j}$($\omega_{i,j} = \frac{v_i+w_j}{2}$-the price of exchange), and ν_i - is the volume of goods, sold by a seller i , y_i - is summ of money,

gained by him. Similarly, z_j - the volume of goods, bought by buyer j ,μ_j - money, spent by him for the purchase of this good.

$\nu_i = \sum_{j=1}^{n} \delta_{i,j}; z_j = \sum_{i=1}^{m} \delta_{i,j}; y_i = \sum_{j=1}^{n} \omega_{i,j}\delta_{i,j}; \mu_j = \sum_{i=1}^{m} \omega_{i,j}\delta_{i,j}.$

If there are several sellers or several buyers with equal minimal or maximal prises, then we consider these several participants as the

one conditional participant (seller or buyer) with united quantity of the goods or with united quantity of money. The money, which this

conditional seller received, distribute between united sellers proportionally their participation in the saling. (Similarly will be with united

buyers). Thus we have the mechanism of market with different prices which was described before.

The above-shown models of single-goods market are dynamic, i.e. all the quantities, introduced above, depend upon time, which we

consider to be discrete, i.e $t = 1, 2, 3, ..,$ as mentioned above. Each period of time consists of three stages.

During the first stage the exchange of goods and money takes place in correspondence with rules, mentioned above.

During the second stage every seller receives the quantity of goods $a_i(t)$, which depends on time,and every buyer receives such a sum of

money $b_j(t)$ (for example it possible $a_i(t) = a_i, b_j(t) = b_j$).

During the third stage each seller and each buyer change minimal and maximal prices.

Formally all, which we wrote before, means the following.

$x_i(t+1) = x_i(t) - \nu_i(t) + a_i(t)$

$u_j(t+1) = u_j(t) - \mu_j(t) + b_j(t)$ (1)

It is assumed, that a_i and b_j satisfy to some conditions

1. The first model of a distribution.If $a_i(t) = a_i$ and $b_j(t) = b_j$ $(i = 1, 2, 3, .., m; j = 1, 2, 3, ..., n)$ are constants, then:

$\sum_{i=1}^{m} a_i = 1; \sum_{j=1}^{n} b_j = 1$

$b_1 \geq b_2 \geq ... \geq b_n, b_j > \frac{\gamma}{2}, b_1 > \frac{1}{n} + \frac{\gamma}{2}$ (2)

2. The second model of a distribution.In this case:

$a_i(t) = \frac{y_i(t)}{\sum_{i=1}^{m} y_i(t)}; i = 1,2,3,...m; b_j(t) = \frac{z_j(t)}{\sum_{j=1}^{n} z_j(t)} j = 1,2,3,...,n$ (3)

where γ can be understood as a minimal possible amount of money (one cent or something similar).

We will examine the rules of changes of maximal and minimal prices of the following kind:

$v_i(t+1) = v_i(t) + \gamma F_\varepsilon(x_i(t), \nu_i(t), x_i(t+1) - x_i(t))\Psi(v_i(t), \frac{y_i(t)}{\nu_i(t)}, F_\varepsilon);$

$w_j(t+1) = w_j(t) - \gamma F_\varepsilon(u_j(t), \mu_j(t), u_j(t+1) - u_j(t))\Phi(\mu_j(t), \frac{z_j(t)}{\mu_j(t)}, F_e);$

The function F_ε, which equals 1 or -1 is the "main function",and Ψ and Φ ,having the value of 0 or 1,are the "functions of carefulness".

$$F_\varepsilon = \begin{cases} 1 \ \ if \ x_i - \epsilon \leq \nu_i \\ -1 \ if \ 0 \leq \nu_i \leq x_i - \varepsilon, x_i(t+1) - x_i(t) \geq 0 \\ 1 \ if \ 0 \leq \nu_i < x_i - \varepsilon, x_i(t+1) - x_i(t) < 0 \end{cases} \quad (4)$$

Similarily,

$$F_\varepsilon = \begin{cases} 1 \ \ if \ u_j - \epsilon \leq \mu_j \\ -1 \ if \ 0 \leq \mu_j \leq u_j - \varepsilon, u_j(t+1) - u_j(t) \geq 0 \\ 1 \ if \ 0 \leq \mu_j < u_j - \varepsilon, u_j(t+1) - u_j(t) < 0 \end{cases} \quad (5)$$

Where ε is enough small constant.

The meaning of the main function F_ε is the following. The seller with index i increases the minimal price on γ ,if he has managed to sell his goods in the moment t. If only a part of his goods was sold, then he increases the price on the value γ ,if at the beginning of $t+1$ moment he has (as a result of the receipt of a_i) less goods than he had in the moment t and he decreases the price on γ in the opposite situation. If buyer with index j has spent just a part of his money, then he decreases his price on the value γ, if by the beginning in the moment $t+1$ as the result of the receipt of b_j , he got less money,than it was in the moment t . Let's mark, that the seller or the buyer may not take part in the exchange in the moment t because minimal price is too big or maximal prise is small size . In this case they have to increase or decrease their prices due to (4) and (5).

During the increase or reducing of prices every participant of the exchange must obey several limitations:

1.All the prices must be higher than 0.

2.Besides when each participant varies his price, he want to protect himself from loss of large number of exchange contacts which was in the previous moment.

The functions of "care " Ψ and Φ are given by following expressions:

$$\Psi = \begin{cases} 0 \quad if \ v_i = 0, F_\varepsilon = -1 \\ 0 \quad if \ \ v_i + \frac{\gamma}{2} \geq \frac{y_i}{\nu_i}, F_\varepsilon = 1 \\ 0 \quad if \ \min\limits_{\delta_{i,j}>0} w_j = v_i, F_\varepsilon = 1 \\ 1 \quad \text{in other cases} \end{cases} \quad (6)$$

$$\Phi = \begin{cases} 0 & if\ w_j = 0, F_\varepsilon = -1 \\ 0 & if\ \ w_j + \frac{\gamma}{2} \geq \frac{\mu_j}{z_j}, F_\varepsilon = 1 \\ 0 & if\ \ {}^{\max v_i = w_j}_{\delta_{i,j} > 0}, F_\varepsilon = 1 \\ 1 & \text{in other cases} \end{cases} \qquad (7)$$

Thanks to Ψ the seller does not increase his minimal price (as needed according to F_ε), if his minimal price v_i in the moment t differs

from the average price of the bargain in the moment t for not more than $\frac{\gamma}{2}$ or, if the last buyer,with whom he deals in the moment t has

the maximal price w_j, which is equal to his minimal v_i, because without fulfilling of these rules he have to search another buyer, whose

changes of prices are not known to him at the present moment.

At the same time,according to function Φ the buyer does not decrease his maximal price w_j (although this is required by F_ε), if his

maximal price w_j in the moment t differs from the average bargain price not more than $\frac{\gamma}{2}$ or at the moment t the last of the sellers, with

whom he deals, has the minimal price v_i is equal to maximal w_j.

Vector with components $x_1(t), x_2(t), ..., x_m(t), v_1(t), v_2(t), .., v_m(t), u_1(t), u_2(t), ..., u_n(t), w_1(t), w_2(t), ..., w_n($ we denote by $r(t)$ and we will

name it the state of the system.

Vector $r(t)$ is stationary, if

$$r(t+1) = r(t) = r^{(0)} \qquad (8)$$

The state, in which all the money are spent and all the goods are sold, i.e.

$$X(t) = Z(t); U(t) = Y(t);$$
$$X(t) = \sum_{i=1}^{m} x_i(t); U(t) = \sum_{j=1}^{n} u_j(t); Y(t) = \sum_{i=1}^{m} y_i(t); Z(t) = \sum_{j=1}^{n} z_j(t) \qquad (9)$$

Let's remark that, if in the moment t the state of market $r(t)$ is a equilibrium then $r(t+1)$ can be not equilibrium. So we are interested the

state of the system, which are simultaneously equilibriums and steady states.

Let us untroduce the following designation:

$$\rho(t) = \max_j w_j(t) - \min_i v_i(t),$$
$$\sigma(t) = \min_j w_j(t) - \max_i v_i(t),$$
$$\lambda(t) = \frac{\max_j w(t) + \min_i v(t)}{2} \qquad (10)$$

The following facts take place for this model.

Theorem 1 *Necessary and sufficient conditions of equilibrium are:*
$v_i(t) = v, w_j(t) = w, i = 1, 2, .., m; j = 1, 2, ..., n; X(t) = Y(t) = Z(t) = U(t) = 1;$

Theorem 2. *At every initial state $r(0)$ of the system there exist such τ , that*
$0 \leq \rho(t) \leq 2\gamma; \sigma(t) \geq 0$ *for* $t > \tau$.

Another mechanism of distribution of goods and money, which enter in the system, between the sellers and buyers according to (3) was considered. Denote by $M\varepsilon$ the set of a steady states of the system.

For the case of the second model of a distributionnt (3) there are theorems.

Theorem 3. *Necessary and sufficient conditions of $r \in M_\varepsilon$ are:*
$w_j = v_i = 1$; $i = 1, 2, ...m$; $j = 1, 2, ...n$;
and one of two conditions:
$X = 1; max_j(u_j - \mu_j \le \varepsilon$ *or* $U = 1; \max_i(x_i - \nu_i) \le \varepsilon$

Theorem 4. *For any initial state of system $r(0)$ there exists such T_1,that for $t > T_1$ we have $r(t) \in M_\gamma$ where $M_\gamma$is determined by following conditions:*
$0 < \sigma < 2\gamma$
$|1 - \lambda| \le \frac{3}{2}\gamma$
$|1 - X| \le 2(3m + 6)\gamma$
$|1 - U| \le 2(3n + 4)\gamma$

The computer modeling of this system with the second model of a distribution showed, that the for any initial state $r(0)$ such T exist that

$r(t) \in M_\varepsilon$ for $t > T$, where accuracy of calculations is γ.

For this model the bankruptcy of the participants is possible, i.e. it is possible that the moment t_s comes, after which for

all $t > t_1$ it will be $r(t) \in M_\varepsilon$ and the number of sellers which has $x_i(t) > \varepsilon$ is m_1 where $m_1 < m$ for $t > t_1$. Also it is possible that the

moment t_2 comes, after which for all $t > t_2$ it will be $r(t) \in M_\varepsilon$. and the number of buyers which has $u_j(t) > \varepsilon$ is $n_1 < n$ for $t > t_2$. It

means that, if at the initial time m sellers and buyers take part in the market, then, as proved by the experiments, since the some moment

there are really only $m_1$sellers and n_1 buyers.

For the case of a_i, b_j are constant (2)(the first model of distribution) there are theorems.

Theorem 5. *The following conditions are neccessary and sufficient for $r \in M_0$:*
$v_i = w_j = 1; i = 1, 2, ..., m; j = 1, 2, ..., n; U = 1; X = 1.$

Theorem 6. *For any initial state $r(0)$ exists such T_2 that $r(t) \in M_\gamma$, if $t > T_2$, where set M_γ satisfies to following conditions:*
$0 \le \sigma \le \rho \le 2\gamma;$
$1 - (\frac{5}{2} + \sqrt{m + 4})\gamma \le \lambda \le 1 + \gamma$
$1 \le X \le 1 + (m + 5)\frac{\gamma}{2}$
$1 \le U \le 1 + (4 + m + 4\sqrt{m + 5})\frac{\gamma}{2}.$

The computer modeling of this system with the first model of a distribution showed, that the for any initial state $r(0)$ such T exist that

$r(t) \in M^{(1)}_\gamma$ for $t > T$, where all $r(t) \in M^{(1)}_\gamma$ satisfy to following conditions:

$1-\gamma \leq \lambda \leq 1+\gamma,$
$1 \leq X \leq 1+\gamma\sqrt{m}, 1 \leq U \leq 1+\gamma\sqrt{n},$
$0 \leq \sigma \leq \rho \leq 2\gamma.$

Thus, in each of represented models of sellers behavior and buyers behaviour the theorems about the existence of the absorbent sets of

steady states which are neighborhood of stationary equilibrium of market and about the global equilibrium of the these sets take place.

This means, that for every initial state the market during the definite time (depending upon the initial state) reaches the neighborhood of

equilibrium. If the model of the participant's activity is (1)-(7) type, (this process was described above) then it is proved that the process

of setting of the equilibrium state consists of the following stages: narrowing of the difference of the prices of sellers and buyers, setting

of the prices near to the level of equilibrium, setting of the equilibrium of flows of goods and money. The fact that the absorbing sets of

state , in which the prices is close to equilibrium prises of all participants, were reached earlier than full equilibrium is reached is very

interesting and may be useful.

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