

## Leviathan group model and its approximation

### ODD protocol

#### Overview

##### Purpose

The Leviathan Model is an opinion dynamics model that combines processes opinion propagation. The model's purpose is to study the evolution of the distribution of opinions in a population of interacting individuals, when each agent belongs to a group. But we also aim at better explaining some patterns generated by this model with a mathematical approximation of the dynamic.

##### Entities, state variables, scales

The model includes  $N_a$  agents and  $n_g$  groups, with  $n_g$  agent in each group. Each agent  $M$  (Me) is an individual who has an opinion  $a_{MY}$  about each agent  $Y$  (You) including herself; the opinions are real values between -1 and +1.  $\sigma$  defines the shape of the propagation function; if  $\sigma$  is very small, the function is very tilted, meaning that agents are subject to high influence from the ones who they evaluate better than themselves and they almost completely disregard the opinions of the ones considered lower. In one time step of the model, we have  $N_a$  interactions of random agent pair. After each step, the opinions of the agents of the same groups are attracted to their average, by a factor  $\mu$  (equal to 1 for no attraction, and 0 for maximal attraction).

##### Process overview, scheduling

In each time step,  $N_a$  interactions occur. For each **interaction**, two random agents are selected to interact. Each agents influence each other and change her opinion about herself and the other agent. At the end of each time step, the opinions of agents of the same groups get closer to their average. The update of opinions is synchronous.

## Design concepts

- Emergence: the distribution of opinions emerge from interactions of pair of agents.
- Sensing: each agent knows each other agent, and can interact with her. The agents have access to a noised opinion of others. Each agent also knows the average opinions of their group.
- Interaction: pairs of individuals interact, and change their respective opinions about each other, and about other agents by means of gossip.
- Stochasticity: the interaction between individuals is a stochastic process because pair are chosen randomly. Moreover, a uniform random noise perturb the evaluation of others' expressed opinions. This noise stands for the inability of an agent to directly access the opinion of another, leading to errors of interpretation.
- Observation: to observe the simulations of the model, we collect at different time step the opinion of each agent about each other agent.

## Initialization

Initially, all opinions are set to 0: agents have a neutral opinion about all the others agents at the beginning of the simulations. However, it is possible to manipulate the starting opinions to conduct some experiment on the model. Anyway, all the opinions about one agent are the same at the initialization.

## Parameters

Parameter		Interval	Reference value
$N_a$	Number of agents	$[2, +\infty]$ , integer	30
$N_g$	Number of agents per group	$[1, +\infty]$ , integer	10
$\delta$	Communication noise	$>0$ , real	0.1
$k$	Number of gossip	$[0, N_a - 2]$ , integer	0 or 5
$\sigma$	Shape of propagation function	$]0, +\infty]$ , real	0.3
$\mu$	Factor of conformity with group	$[0, 1]$ , real	$1 - 0.005/N_g$

## Interaction

During interactions, Y express their self-opinion  $a_{YY}(t)$  and their opinion  $a_{YM}(t)$  about agent M. When M gets the message  $a_{YY}(t)$  about Y's self-opinion, her opinion  $a_{MY}(t)$  about Y is modified and gets closer to a noisy evaluation of  $a_{YY}(t)$ . The modification of  $a_{MY}(t)$ , denoted by  $\Delta a_{MY}(t)$ , is ruled by the following equation, in which  $R(\delta)$  designates a uniformly drawn number between  $-\delta$  and  $\delta$ .

$$\Delta a_{MY}(t) = p_{MY}(t)(a_{YY}(t) - a_{MY}(t) + R(\delta))$$

Similarly, the change of opinions  $a_{MM}(t)$  is given by:

$$\Delta a_{MM}(t) = p_{MY}(t)(a_{YM}(t) - a_{MM}(t) + R(\delta))$$

The function of influence  $p_{MY}(t)$  is the following:

$$p_{MY}(t) = \frac{1}{1 + \exp\left(\frac{a_{MM}(t) - a_{MY}(t)}{\sigma}\right)}$$

Overall, after the encounter between M and Y, the opinions of M change as follows:

$$a_{MM}(t + 0.5) = a_{MM}(t) + \Delta a_{MM}(t)$$

$$a_{MY}(t + 0.5) = a_{MY}(t) + \Delta a_{MY}(t)$$

The opinions of Y change similarly (inverting Y and M in the equations). The opinions are always between -1 and +1.

When activating gossip ( $k > 0$ ), agent Y also talks about  $k$  agents H drawn at random and the changes of the opinion of M about agents H are:

$$\Delta a_{MH}(t) = p_{MY}(t)(a_{YH}(t) - a_{MH}(t) + R(\delta))$$

Similarly, agent M talks to Y about  $k$  agents drawn at random and the changes of the opinion of Y about these agents follow the same equations where Y and M are inverted.

The Opinions of M and Y are updated synchronously: at each encounter, all the changes of opinions are first computed, and then the opinions are modified simultaneously.

### Group conformity

After the interactions, each opinion is attracted by the average opinion of the agents of the same group. For all agent  $i$  and  $j$ :

$$a_{ii}(t+1) = \mu \cdot a_{ii}(t+0.5) + \frac{1-\mu}{N_g} \sum_{p \in I} a_{pp}(t+0.5)$$

and

$$a_{ij}(t+1) = \mu \cdot a_{ij}(t+0.5) + \frac{1-\mu}{N_g(N_g - \delta_{IJ})} \sum_{p \in I} \sum_{q \in J, p \neq j} a_{pq}(t+0.5)$$

With  $I$  is the group of  $i$ ,  $J$  the group of  $j$ , and  $\delta_{IJ}$  equal to 1 if and only if  $I=J$ .