Chapter 3

The Model

In this chapter we describe both the general aspects of the model that are common to all experimental treatments and what differentiates each optimisation strategy and regulatory framework.

The experimental design presents an overview of all different experimental treatments used to investigate the consequences of implementing the Basel III regulatory framework. The visualisation of all treatments, the description of their main characteristics and distinguishable features facilitate the understanding of the entire simulation and the relation between all its components.

The description of the model market structure identifies the constitutive parts of the model and how agents interact within the model framework. This model permits to understand whether the capital adequacy regulation represents a source of increased risk or an adequate response to mitigate risk and the consequences of future financial crisis. The potential homogenisation of banks' behaviour and subsequent increase in dediversification effects, market instability and increased risk is investigated using an ABM adapted from existing models in the literature (Chiarella and Iori [2002] and Chiarella et al. [2009]). This approach consists of modelling financial markets as a population of agents identified by their decision rules, which can be considered as a mapping from agents' information set to the set of possible actions: buy, sell or hold. Lastly, we identify the parameters settings for each treatment of the model and describe the *rationale* behind their choice.

In the next sections we describe the structural elements of the model, which are common to both agents' optimisation strategies covered in chapters 5 and 6. In section 3.1 we present the experimental design of our model. In section 3.2 we identify the market structure over which our model evolves and describe the algorithms of all different treatments. In section 3.3 we present the parameters settings for all treatments. Lastly, in section 3.4 we conclude.

3.1 Experimental Design

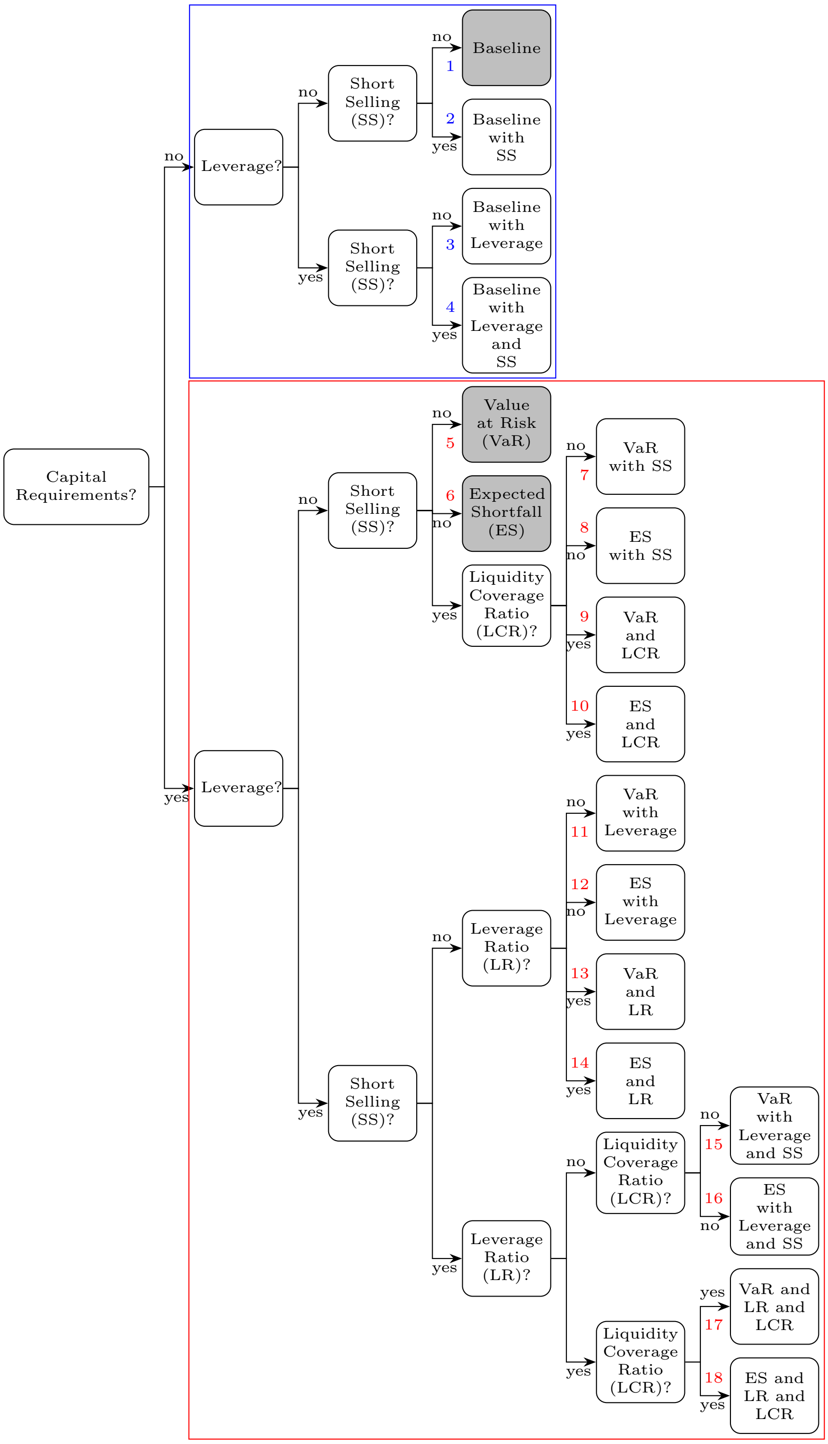
In order to test our hypothesis and answer to the research questions of whether the capital requirements regulation reduces individual banks' riskiness and consequently market instability, or if inadvertently increases banks' portfolio dediversification and contagion effects, our experiments are analysed using two general experimental treatments: with and without financial regulation. The treatment without regulation represents the control group and its results are used to juxtapose to the results of our treatment groups and, thus, conclude if the introduction of regulation has a positive or negative impact on bank's behaviour. In order to test the effects of portfolio optimisation strategies the experimental design is divided in two distinct optimisation strategies: Modern Portfolio Theory and Cumulative Prospect Theory.

Different experimental treatments, or sub-treatments, are considered and each implements different risk metrics, or combinations of risk metrics. In experimental treatments where agents cannot leverage nor short-sell we only implement VaR and ES. When leverage is introduced we investigate the combined impact of the leverage ratio, VaR and ES on banks' behaviour, and in experimental treatments with short-selling we add the liquidity coverage ratio to VaR and ES to conclude about the impact of this regulatory feature. In the experimental treatment where both leverage and short-selling are considered, then the joint impact of LR and LCR is investigated along with VaR and ES.

Figure 3.1 shows how we administer these treatments on a group of agents to observe their response to the introduction of a regulatory framework, or, alternatively, to changes to this framework.

The distinction between treatments with and without regulation is defined by the red and blue boxes, respectively. The existence of financial regulation is

Figure 3.1: Experimental Design



Note: The tree describes the experimental design for treatments using Modern Portfolio Theory. The shaded nodes represent treatments using Cumulative Prospect Theory. The blue (red) box represents the experimental treatments without (with) regulation for both optimisation strategies.

determined by the implementation of minimum risk-based capital requirements, as the regulation on leverage and liquidity are subsequent stages of the regulatory framework and implemented as an extension of risk-based capital requirements regulation. According to BCBS [2011] the leverage ratio and the liquidity coverage ratio represent a complementary aspect of the minimum risk-based capital requirements regulation.

In the case of absence of minimum capital requirements, and consequently of financial regulation, the treatments within the blue box represent the control groups of our experiments (nodes 1 to 4). The results obtained in these cases are then compared to the results obtained from the experimental treatments, where financial regulation is implemented (nodes 5 to 18). This comparative analysis allows us to confront the impact of experimental treatments, with and without financial regulation, on banks' behaviour.

The first experimental treatment identifies the existence, or absence, of risk-based minimum capital requirements, which determines the implementation, or absence, of regulation. If financial regulation exists, then agents have to adapt their behaviour to a mandatory minimum risk-based capital requirement by applying VaR or ES as a market risk metric. If leverage is permitted then agents can leverage their positions and should keep a minimum leverage ratio if leverage regulation is implemented. If short-selling is allowed then agents can have negative quantities of stock and should adapt their behaviour according to a mandatory minimum liquidity ratio, if liquidity regulation exists.

The comparison of the results obtained from these different experimental treatments allows to investigate the hypothesis of whether Basel III framework and capital requirements regulation lead to portfolio dediversification and subsequent destabilisation of financial markets or, alternatively, if the financial markets exhibit greater resilience. Our model identifies these known or expected sources of variability in agents' behaviour, but to reduce the effect of other sources of variability in the comparison of results, e.g. initial conditions, which could affect the accuracy of our answers to the research questions, we implement a model where comparable treatments share the same initial conditions and free parameters remain constant. This procedure guarantees that the initial conditions are identical, which eliminates the effect of these potential sources of variability, and consequently allows to derive more robust comparisons between treatments.

3.2 Model Market Structure

We use an ABM of a financial market in which heterogeneous agents can invest in both risky and risk-free assets (e.g. Brock and Hommes [1998], Chiarella and Iori [2002], Chiarella et al. [2009], Hermsen [2010], Thurner [2012]). We use an agent-based model with risk-free and risky assets to investigate the impact of the regulatory framework on both banks' portfolio management and market behaviour. If agents only consider their demand for shares in isolation, in a single-asset model, without modelling the agents' wider portfolio optimisation problem and risk management strategy, the model wouldn't be suitable for exploring the implications of Basel III since agents wouldn't balance their capital against risk-weighted assets.

The ABM here presented consists of a population of agents, in our case financial institutions, n_a, trading in an order-driven market with continuous clearing, over a period of time corresponding to two years, with no official market maker, in which orders are submitted in a double auction and executions follow price/time priority.

We restrict our world to one in which financial institutions construct a portfolio consisting of two assets: a risky asset, stocks, and a risk-free asset, cash. Therefore, we use equity positions as a proxy for market risk factors. Financial institutions are considered to be risk sensitive which makes them rebalance their portfolio every time they place an order in the market. All financial institutions have heterogeneous expectations about the expected returns and transaction costs and taxes are assumed to be zero.

Financial institutions can post two types of orders: buy or sell. Every time a financial institution i is chosen to enter the market this financial institution i can submit a limit order, an order to trade a certain quantity of stocks at a given price. These orders are submitted sequentially to an electronic trading system, matched and executed automatically. This is known as the limit-order book, where the lowest price for which there is an outstanding limit sell order, which is called the ask price, matches the highest buy price, which is called the bid price.

If agents submit an order before their previous order gets executed, the latest order works as a cancellation order and overrides the previous one.

In our model agents can place orders of size larger than one which allow us to explore the implications of regulatory proposals, such as Basel III, for portfolio dediversification and market instability.

Each financial institution receives an initial endowment of cash, c_0^i, and an initial quantity of stocks, s_0^i. All agents know the fundamental price, p_t^f, which follows a geometric Brownian motion (GBM), as in Chiarella et al. [2009]:

$$\frac{\Delta S}{S} = \mu \Delta t + \sigma \epsilon \sqrt{\Delta t} \tag{3.1}$$

where ΔS is the change in the stock price S in a small time interval Δt and ϵ has a standard normal distribution. The parameter μ is the drift and σ is the volatility of the fundamental price.

The price at time t, p_t, is determined by the market and is given by the price at which transactions occur. If no transactions occur at a given moment in time then the price is determined by the last transaction price. If no bids or asks are listed in the book then a proxy of the price is given by the previous traded or quoted price. The risk-free rate, r_f, is assumed to be constant over time and the same for all agents.

Despite the fact that we investigate the potential occurrence of defaults, in our model there is no actual default, which means that agents stay in the market even if they cannot participate due to technical default, i.e. when they fail to: 1) fulfil an obligation to repay a loan in case of leverage, or 2) buy-back the stock at some point in the future in case of short-selling. In a situation of technical default agents stay in the market, even if they cannot temporarily participate, as a potential increase in stocks prices can generate positive changes in agents' balance sheet and put agents actively back into the market. This possible scenario shows the importance of oscillations in the balance sheet, even in the absence of trading, and the endogenous risk (Shin [2010], Beale et al. [2011], Zhou [2013]). In our model there is no lending/borrowing between financial institutions, which means that any systemic effect we might see in the model cannot be attributed to financial networks or interconnections. Instead, spillover effects operate through financial institutions behaviours and impact on market prices, rather than direct exposure between them.

3.2.1 Financial Institutions' Expectations

Economic agents form expectations and act on the basis of predictions generated by these expectations (Arthur et al. [1997]). Agents' intrinsic strategies are partially modelled based on their expectations of future prices and consist of three components: fundamentalist, chartist and noise-induced. Financial institution i time horizon, τ^i, depends on its components. Long term investors typically give more weight to fundamentalist strategies with longer time horizons, whilst day traders give more weight to chartist rules. Hence, the time horizon is a function of the probability of each agent entering the market, λ^i, and determines the interval $(t + \tau(\lambda^i))$ while the agent's expectation about the return will prevail.

Every time an agent i is chosen to enter the market, this financial institution i forms an expectation in time t about the return in time $t + \tau(\lambda^i)$, $\widehat{r}^i_{t,t+\tau(\lambda^i)}$. Financial institutions make their expectations about returns based on the following equation:

$$\widehat{r}^i_{t,t+\tau(\lambda^i)} = g^i_1 \log(\frac{p^f_t}{p_t}) + g^i_2 \overline{r}_{t,L^i} + n^i \epsilon^i_t \tag{3.2}$$

where g^i_1, g^i_2 and n^i represent the weights given to fundamentalist, chartist and noise-induced components, respectively. The sign of g^i_2 indicates a trend chasing strategy if $g^i_2 > 0$ and a contrarian if $g^i_2 < 0$. All financial institutions use a linear combination of these components.

The fundamentalist component is assumed to have a stabilising effect on prices, whereas the chartist component has the opposite effect and tends to have a destabilising effect generating large price jumps and driving asset prices away from the intrinsic value of the asset. The average return over the interval used by the chartist component is given by

$$\overline{r}_{t,L^i} = \frac{1}{L^i} \sum_{j=1}^{L^i} \log \frac{p_{t-j}}{p_{t-j-1}}. \tag{3.3}$$

L^i is uniformly and independently distributed across financial institutions over the interval $(1, L_{max})$.

The noise component is randomly assigned across financial institutions, $\epsilon_t^i \sim N(0,1)$.

The price expected at $t + \tau(\lambda^i)$ by financial institution i is given by

$$\widehat{p}_{t,t+\tau(\lambda^i)}^i = p_t e^{\widehat{r}_{t,t+\tau(\lambda^i)}^i} \tag{3.4}$$

3.2.2 Model Constraints

Financial institutions' wealth is constituted by cash and stocks and all financial institutions are given an initial endowment of cash and stocks. Thus the wealth expression for financial institution i at time t is represented by:

$$W_t^i = c_t^i + s_t^i \times p_t \tag{3.5}$$

where c_t^i represents the amount of cash, s_t^i the quantity of stocks and p_t the current price. If equation 3.5 is negative then agent i is in technical default.

Financial institutions' behaviour can be restricted by two types of constraints: a budget constraint and/or regulatory constraints, depending on the treatment. In the next sections we identify these two types of constraints and how they affect financial institutions' behaviour.

3.2.2.1 Budget Constraint

What determines the optimal demand for assets in investors' portfolios depends on how the maximisation problem is set up, as described in sections 2.2.2.1, 2.2.3.1, 5.1 and 6.1, subject to the investor's constraints. When leverage is not allowed, which is represented by a maximum leverage of 1, all agents' trading is limited by a budget constraint. However, when leverage or short-selling are permitted, agents can choose an optimal proportion of the risky asset above 1 or below 0, respectively. In the next section we explain how this budget constraint influences agents' trading behaviour.

3.2.2.1.1 Portfolio Selection without Regulation

Once the financial institution has formed its own expectations of the future price and solved the problem of the optimal share of the risky asset, the financial institution enters the market by placing a buy or a sell order subject to a budget constraint, or holds trade, which means that the bank does not place an order. If the optimal quantity of stocks is greater (lower) than the current amount of stocks, then the financial institution is willing to buy (sell) at the following price:

$$\text{buy: } b_t^i = \widehat{p}^i_{t+\tau(\lambda^i)} - (\widehat{p}^i_{t+\tau(\lambda^i)})^{\kappa^i} \tag{3.6}$$

$$\text{sell: } a_t^i = \widehat{p}^i_{t+\tau(\lambda^i)} + (\widehat{p}^i_{t+\tau(\lambda^i)})^{\kappa^i} \tag{3.7}$$

where κ^i is randomly assigned across agents using a negative exponential distribution.

The optimal quantity of stocks is represented by

$$q_t^{i*} = \frac{W_t^i}{p_t} \times y_t^{i*} \tag{3.8}$$

where y_t^{i*} is the optimal share of stocks. This optimal quantity is calculated before any order is posted or any transaction occurs, either buy or sell. In each case, the optimal quantity for buy and sell is, respectively, $q_{b,t}^i$ and $q_{a,t}^i$:

$$q_{b,t}^{i*} = \frac{W_t^i}{b_t^i} \times y_t^{i*} \tag{3.9}$$

$$q_{a,t}^{i*} = \frac{W_t^i}{a_t^i} \times y_t^{i*} \tag{3.10}$$

However, the order is placed if and only if the following conditions are met:

Baseline treatment without leverage nor short-selling

The required variables to place an order in the baseline treatment without leverage nor short-selling are the following: the optimal quantity of stocks, q_t^{i*}, the current funds, c_t^i, and the current amount of stocks, s_t^i. Funds and stocks should be non-negative as there is no leverage nor short-selling. The algorithm ensures that a decision regarding q_t^i should be taken as the output.

Algorithm 1 Baseline portfolio optimisation trading algorithm

Require: $q_t^{i*} \wedge c_t^i \geq 0 \wedge s_t^i \geq 0$
Ensure: q_t^i
  if $q_t^i > s_t^i$ **then**
    buy: $q_{b,t}^i \leftarrow \min(q_{b,t}^{i^*} - s_t^i, \gamma_t^i)$; price $\leftarrow b_t^i$
  else if $q_t^i < s_t^i$ **then**
    sell: $q_{a,t}^i \leftarrow s_t^i - q_{a,t}^{i^*}$; price $\leftarrow a_t^i$
  else
    hold
  end if

For agents to place a buy order the optimal quantity of stocks should be greater than the current amount of stocks as follows:

$$q_{b,t}^{i*} > s_t^i \tag{3.11}$$

In experiments without leverage, agents' optimal share of stocks is subject to the following budget constraint:

$$\gamma_t^i = \frac{c_t^i}{b_t^i} > 1 \tag{3.12}$$

The volume of the buy order is a function of these conditions:

$$q_{b,t}^i = \min(q_{b,t}^{i*} - s_t^i, \gamma_t^i). \tag{3.13}$$

If the optimal amount of stocks is smaller than the current amount of stocks

$$q_{a,t}^{i*} < s_t^i \tag{3.14}$$

then the financial institution i is willing to sell at price a_t^i the following quantity of stocks:

$$q_{a,t}^i = s_t^i - q_{a,t}^{i*} \tag{3.15}$$

As there is no short-selling in this treatment, equation 3.8 is never negative and the amount of stocks the agent is willing to sell is the difference between the current and the optimal amount of stocks. Therefore, from equations 3.14 and 3.15, the volume of the sell order is always non-negative. Lastly, under any other conditions the financial institution holds trade.

The trading conditions in the baseline treatment are shown in algorithm 1.

Baseline treatment with leverage

In the baseline treatment with leverage, the final quantity of stocks is not subject to a liquidity constraint (v. equation 3.12). The maximum proportion that the risky asset, y_t^{i*}, can assume in equation 3.8 is 60 (BoE [2012]). The choice of 60 as a ceiling for leverage is explained in more detail in section 3.3. The final volume of the buy order is a function of the equation 3.11 and represented as follows:

$$q_{bLev,t}^{i} = q_{b,t}^{i^*} - s_t^i \tag{3.16}$$

Hence, in this treatment the required variables are the optimal quantity of stocks and a non-negative current amount of stocks. The selling conditions are exactly the same as in the simpler baseline scenario (v. equation 3.15), therefore the volume of the sell order is represented by:

$$q_{aLev,t}^{i} = q_{a,t}^{i} = s_t^i - q_{a,t}^{i^*} \tag{3.17}$$

Otherwise, the financial institution holds trade. Algorithm 2 summarises the trading conditions for the baseline treatment with leverage.

Baseline treatment with short-selling

The liquidity constraint in equation 3.12 is implemented in treatments without leverage, as agents cannot borrow cash to invest in risky assets. Hence, in treatments where only short-selling is allowed this liquidity constraint imposes a threshold on agents' portfolio decisions.

In treatments with short-selling but no leverage, for agents to place a buy order the optimal quantity of stocks should be greater than the current amount

Algorithm 2 Baseline portfolio optimisation trading algorithm with leverage

Require: $q^{i*}_{Lev,t} \wedge s^i_t \geq 0$
Ensure: $q^i_{Lev,t}$
  if $q^{i*}_{Lev,t} > s^i_t$ **then**
    buy: $q^i_{bLev,t} \leftarrow q^{i*}_{Lev,t} - s^i_t$; price $\leftarrow b^i_t$
  else if $q^{i*}_{Lev,t} < s^i_t$ **then**
    sell: $q^i_{aLev,t} \leftarrow s^i_t - q^{i*}_{Lev,t}$; price $\leftarrow a^i_t$
  else
    hold
  end if

of stocks, as in the baseline scenario (v. equation 3.13):

$$q^i_{bSS,t} = q^i_{b,t} = \min(q^{i*}_{b,t} - s^i_t, \gamma^i_t) \tag{3.18}$$

If the optimal amount of stocks is smaller than the current amount of stocks then the financial institution i is willing to sell at price a^i_t the following quantity of stocks:

$$q^i_{aSS,t} = s^i_t - q^{i*}_{a,t} \tag{3.19}$$

Contrarily to previous treatments, with short-selling $q^{i*}_{a,t}$ can be negative and, consequently, the final amount of stocks can also be negative. Lastly, under any other conditions the financial institution holds trade. Algorithm 3 summarises the above conditions.

Algorithm 3 Baseline portfolio optimisation trading algorithm with short-selling

Require: $q^{i*}_{SS,t} \wedge c^i_t \geq 0$
Ensure: $q^i_{SS,t}$
  if $q^{i*}_{SS,t} > s^i_t$ **then**
    buy: $q^i_{bSS,t} \leftarrow \min(q^{i*}_{SS,t} - s^i_t, \gamma^i_t)$; price $\leftarrow b^i_t$
  else if $q^{i*}_{SS,t} < s^i_t$ **then**
    sell: $q^i_{aSS,t} \leftarrow s^i_t - q^{i*}_{SS,t}$; price $\leftarrow a^i_t$
  else
    hold
  end if

Baseline treatment with leverage and short-selling

In treatments with leverage and short-selling, agents can borrow cash and the amount of stocks in agents' portfolio can be negative. Hence, the required constraints of previous treatments are not present in this case. The amount of stocks placed in the order-book is the following to buy and sell orders, respectively:

$$q^{i}_{bLevSS,t} = q^{i^*}_{b,t} - s^{i}_{t} \tag{3.20}$$

and

$$q^{i}_{aLevSS,t} = s^{i}_{t} - q^{i^*}_{a,t} \tag{3.21}$$

The buying conditions replicate the treatment with leverage and the selling conditions replicate the treatment with short-selling. Lastly, under any other conditions the financial institution holds trade. Algorithm 4 summarises the above conditions.

Algorithm 4 Baseline portfolio optimisation trading algorithm with leverage and short-selling

Require: $q^{i^*}_{LevSS,t}$
Ensure: $q^{i}_{LevSS,t}$
  if $q^{i^*}_{LevSS,t} > s^{i}_{t}$ **then**
    buy: $q^{i}_{bLevSS,t} \leftarrow q^{i^*}_{LevSS,t} - s^{i}_{t}$; price $\leftarrow b^{i}_{t}$
  else if $q^{i^*}_{LevSS,t} < s^{i}_{t}$ **then**
    sell: $q^{i}_{aLevSS,t} \leftarrow s^{i}_{t} - q^{i^*}_{LevSS,t}$; price $\leftarrow a^{i}_{t}$
  else
    hold
  end if

3.2.2.2 Regulatory Constraints

In addition to leverage and short-selling constraints we implement regulatory constraints (v. section 2.1.5.3). These regulatory constraints restrict the limits of agents' behaviour and allow us to evaluate the impact of capital requirements on portfolio dediversification and market stability. These regulatory constraints can assume the form of risk-based capital requirements, leverage ratio and liquidity coverage ratio. The regulatory constraints are implemented as regulatory layers. Hence, LR and LCR are implemented in addition to VaR and ES regulatory constraints, as additional and more stringent regulatory layers. These are summarised in turn below.

3.2.2.2.1 Value-at-Risk

In our simulation the previous day VaR number is estimated using historical simulation. Data are collected on movements in the artificial stock market which provides N alternative scenarios for what can happen between today and tomorrow, where N is the considered number of completed days at the moment of the VaR estimation. For each scenario, the change in the value of the portfolio between today and tomorrow is calculated. This defines a probability distribution for daily losses (gains are considered negative losses) in the value of agents' portfolio. The value of the portfolio under i^{th} scenario is calculated as follows (Hull [2012]):

$$V^{i^{th}} = v_n \frac{v_i}{v_{i-1}} \tag{3.22}$$

where v_i is the value of the risky assets on day i, and today is represented as day n.

Scenario i assumes that the percentage changes between today and tomorrow are the same as they were between day $i-1$ and i for $1 \leq i \leq 252$ in case of VaR, or $1 \leq i \leq 504$ for ES.

The losses for all the scenarios are then ranked in descending order and the one-day 99 percent VaR can be estimated as the n^{th}-worst loss over a maximum of 252 days for VaR or 504 days for ES. The 10-day VaR is then calculated as $\sqrt{10}$ times the one-day 99 percent VaR:

$$\text{10-day VaR}_t^i = \text{1-day VaR}_t^i \times \sqrt{10} \tag{3.23}$$

The sVaR is calculated daily and calibrated to the existent historical data at the time of the calculation. Hence, sVaR is calculated based on the n^{th}-worst loss observed since the beginning of the simulation, i.e. over a maximum of 504 days or the available data at that moment in time. The results of the calculations of VaR and sVaR are scaled up, respectively, by the multiplication factors $m^i_{c,t}$ or $m^i_{s,t}$. We use the conventional 252 days period (when existent) in the case of VaR and all the available data at the moment of the calculation in the case of sVaR and ES to capture all the periods of significant financial stress.

We implement a backtesting daily assessment on actual, not hypothetical, changes in the portfolio's value based on a comparison between the portfolio's end-of-day value and its actual value at the end of the subsequent day excluding fees, commissions, and net interest income (BCBS [2006], Rossignolo et al. [2013]):

$$(P_t - P_{t-1} \times S_t) \tag{3.24}$$

The boundaries of the backtesting results are deduced by calculating the binomial probabilities of obtaining a particular number of exceptions from a sample of 252 independent observations, associated with an accurate model with true level of coverage of 99 percent. Under these assumptions, we calculate the probability of obtaining exactly a certain number of exceptions in a sample of 252 independent observations. When there are no 252 observations available, the backtesting results calculate the binomial probability of obtaining a certain number of exceptions from a sample of the number of days in the simulation at that moment in time.

Over a trading day institutional agents compare their previous end of day value-at-risk (VaR^i_{t-1}) with their current cash position. If the capital requirements constraints are violated then institutional investors must sell the necessary amount of their risky stock during the trading day in order to meet the minimum capital requirements.

Institutional agents place orders if and only if the following conditions are met:

VaR treatments

Under regulatory constraints, the previous liquidity constraint (v. equation 3.12) becomes also a function of the minimum capital requirements constraint:

$$\gamma^i_{VaR,t} = \frac{c^i_t - k^i_{VaR,t}}{b^i_t} > 1 \tag{3.25}$$

For this equation to be greater than 1 the following condition is necessary, although not sufficient:

$$k^i_{VaR,t} < c^i_t$$

If the optimal portfolio optimisation determines a buy order, $q^{i*}_{b,t} > s^i_t$, therefore the volume of the buy order with VaR constraint is the following:

$$q^i_{bVaR,t} = \min(q^{i^*}_{b,t} - s^i_t, \gamma^i_{VaR,t}) \tag{3.26}$$

However, if the portfolio optimisation without capital requirements determines that $q^{i^*}_t < s^i_t$, than the buying condition under VaR constraint is ignored and substituted by a sell order as in equation 3.15. Otherwise, agents' hold trade.

If the minimum capital requirements are greater than the current amount of funds or if the optimal amount of stocks is smaller than the current amount of stocks,

$$k^i_{VaR,t} > c^i_t \tag{3.27}$$

or

$$q^{i^*}_{a,t} < s^i_t \tag{3.28}$$

then the financial institution i places a sell order. However, this order is placed differently depending on whether equation 3.27 or equation 3.28 are verified.

The regulatory condition in equation 3.27 is given priority over the portfolio optimisation condition in equation 3.28, as the financial institution has to fulfil the regulatory own funds requirement (v. equation 2.1) at the end of each day. Hence, a market order, an order to immediately buy or sell a certain quantity of stocks at the best available opposite quote, is placed to sell the necessary quantity of the risky asset at the best available price in the limit-order book. When a market order arrives it is matched with the best available price in the limit order-book, and a trade occurs at price:

$$\text{sell: } a^i_{VaR,t} = \max(b_t) \tag{3.29}$$

and the final quantity of stocks to sell is the following:

$$q^i_{aVaR,t} = \min(\frac{k^i_{VaR,t} - c^i_t}{a^i_{VaR,t}}, s^i_t) \tag{3.30}$$

where $k^i_{VaR,t} - c^i_t$ must be greater than 0 to ensure that the selling threshold defined by VaR conditions is not violated, and in treatments without short-selling the current amount of stock, s^i_t, must be the maximum quantity of stocks that agents can sell.

If equation 3.28 is verified but not 3.27, which reflects the portfolio optimisation condition without regulatory constraint, a limit order is placed in the order-book, as in the selling conditions of the baseline treatment. In the case where

$$k^i_{VaR,t} = c^i_t \wedge q^{i^*}_{a,t} < s^i_t$$

then agents follow the portfolio optimisation and place a sell order. Under any other conditions the financial institution holds trade. Algorithm 5 summarises the conditions described above.

We conclude that from the implementation of risk-based capital requirements regulation, agents' portfolio maximisation is constrained by the introduction of $k^i_{VaR,t}$, either in selling or buying orders.

The capital requirements algorithms with leverage and short-selling have to be extended to reflect the required variables from algorithms 2 and 3, respectively and, consequently, the treatment with leverage and short-selling from algorithm 4. Thus, in algorithms 6, 7 and 8, the required variables from capital requirements, $c^i_t - k^i_t \geq 0$, and the required variables from leverage and short-selling treatments are combined to form the condition of the algorithms for the respective treatments.

In the treatment with VaR and leverage, $c^i_t \geq 0$ is not required as agents can leverage their positions. Similarly, in the treatment with short-selling $s^i_t \geq 0$ is not required as negative stocks positions are allowed. These two conditions are then

Algorithm 5 Portfolio optimisation trading algorithm with VaR

Require: $q^{i*}_{VaR,t} \wedge c^i_t \geq 0 \wedge s^i_t \geq 0 \wedge c^i_t - k^i_t \geq 0$
Ensure: $q^i_{VaR,t}$
  if $k^i_t < c^i_t$ **then**
    if $q^i_t \leftarrow q^i_{b,t}$ **then**
      buy: $q^i_{bVaR,t} \leftarrow \min(q^{i*}_{b,t} - s^i_t, \gamma^i_{VaR,t})$; price $\leftarrow b^i_t$
    else if $q^i_t \leftarrow q^i_{a,t}$ **then**
      sell: $q^i_{aVaR,t} \leftarrow s^i_t - q^{i*}_{a,t}$; price $\leftarrow a^i_t$
    else
      hold
    end if
  else if $k^i_t > c^i_t$ **then**
    sell: $q^i_{aVaR,t} \leftarrow \min(\frac{k^i_{VaR,t} - c^i_t}{a^i_{VaR,t}}, s^i_t)$; price $\leftarrow a^i_{VaR,t}$
  else if $k^i_t = c^i_t \wedge q^{i*}_t < s^i_t$ **then**
    sell: $q^i_{aVaR,t} \leftarrow s^i_t - q^{i*}_{a,t}$; price $\leftarrow a^i_t$
  else
    hold
  end if

jointly implemented in the treatment with VaR with leverage and short-selling.

Hence, in trading algorithms with regulatory capital requirements, agents' optimal portfolio maximisation, derived from their portfolio maximization function, is constrained by the minimum capital requirement calculated at the end of each day.

3.2.2.2.2 Expected Shortfall

According to the stressed metric for minimum capital requirements recommended by the BCBS (v. equation 2.2), in our model ES is calibrated to all observed data, until the maximum of a two years period, which consequently captures the most severe stress periods.

The one-day 97.5 percent ES is estimated as the average of the n-worst losses (v. equation 3.22), and the 10-day ES is then calculated as $\sqrt{10}$ times the one-day 97.5 percent ES:

Algorithm 6 Portfolio optimisation trading algorithm with VaR and leverage

Require: $q^{i*}_{VaRLev,t} \wedge s^i_t \geq 0 \wedge c^i_t - k^i_t \geq 0$
Ensure: $q^i_{VaRLev,t}$
  if $k^i_t < c^i_t$ **then**
    if $q^i_{Lev,t} \leftarrow q^i_{bLev,t}$ **then**
      buy: $q^i_{bVaRLev,t} \leftarrow \min(q^{i*}_{b,t} - s^i_t, \gamma^i_{VaR,t})$; price $\leftarrow b^i_t$
    else if $q^i_{Lev,t} \leftarrow q^i_{aLev,t}$ **then**
      sell: $q^i_{aVaRLev,t} \leftarrow s^i_t - q^{i*}_{a,t}$; price $\leftarrow a^i_t$
    else
      hold
    end if
  else if $k^i_t > c^i_t$ **then**
    sell: $q^i_{aVaRLev,t} \leftarrow \min(\frac{k^i_{VaR,t} - c^i_t}{a^i_{VaR,t}}, s^i_t)$; price $\leftarrow a^i_{VaR,t}$
  else if $k^i_t = c^i_t \wedge q^{i*}_t < s^i_t$ **then**
    sell: $q^i_{aVaRLev,t} \leftarrow s^i_t - q^{i*}_{a,t}$; price $\leftarrow a^i_t$
  else
    hold
  end if

Algorithm 7 Portfolio optimisation trading algorithm with VaR and short-selling

Require: $q^{i*}_{VaRSS,t} \wedge c^i_t - k^i_t \geq 0$
Ensure: $q^i_{VaRSS,t}$
  if $k^i_t < c^i_t$ **then**
    if $q^i_{SS,t} \leftarrow q^i_{bSS,t}$ **then**
      buy: $q^i_{bVaRSS,t} \leftarrow \min(q^{i*}_{b,t} - s^i_t, \gamma^i_{VaR,t})$; price $\leftarrow b^i_t$
    else if $q^i_{SS,t} \leftarrow q^i_{aSS,t}$ **then**
      sell: $q^i_{aVaRSS,t} \leftarrow s^i_t - q^{i*}_{a,t}$; price $\leftarrow a^i_t$
    else
      hold
    end if
  else if $k^i_t > c^i_t$ **then**
    sell: $q^i_{aVaRSS,t} \leftarrow \frac{k^i_{VaR,t} - c^i_t}{a^i_{VaR,t}}$; price $\leftarrow a^i_{VaR,t}$
  else if $k^i_t = c^i_t \wedge q^{i*}_t < s^i_t$ **then**
    sell: $q^i_{aVaRSS,t} \leftarrow s^i_t - q^{i*}_{a,t}$; price $\leftarrow a^i_t$
  else
    hold
  end if

Algorithm 8 Portfolio optimisation trading algorithm with VaR, leverage and short-selling

Require: $q^{i*}_{VaRLevSS,t} \wedge c^i_t - k^i_t \geq 0$
Ensure: $q^i_{VaRLevSS,t}$
  if $k^i_t < c^i_t$ **then**
    if $q^i_{LevSS,t} \leftarrow q^i_{bLevSS,t}$ **then**
      buy: $q^i_{bVaRLevSS,t} \leftarrow \min(q^{i*}_{b,t} - s^i_t, |\gamma^i_{VaR,t}|)$; price $\leftarrow b^i_t$
    else if $q^i_{LevSS,t} \leftarrow q^i_{aLevSS,t}$ **then**
      sell: $q^i_{aVaRLevSS,t} \leftarrow s^i_t - q^{i*}_{a,t}$; price $\leftarrow a^i_t$
    else
      hold
    end if
  else if $k^i_t > c^i_t$ **then**
    sell: $q^i_{aVaRLevSS,t} \leftarrow \frac{k^i_{VaR,t} - c^i_t}{a^i_{VaR,t}}$; price $\leftarrow a^i_{VaR,t}$
  else if $k^i_t = c^i_t \wedge q^{i*}_t < s^i_t$ **then**
    sell: $q^i_{aVaRLevSS,t} \leftarrow s^i_t - q^{i*}_{a,t}$; price $\leftarrow a^i_t$
  else
    hold
  end if

$$\text{10-day ES}^i_t = \text{1-day ES}^i_t \times \sqrt{10} \tag{3.31}$$

We use a discrete version of the ES as suggested by the Basel Committee (BCBS [2013]) and we will not focus our attention on the discussion whether ES is an appropriate risk measure or if a continuous version of the ES is preferential to the discrete one (e.g. Acerbi and Tasche [2002], Acerbi and Szekely [2014], Chen [2014]). Our only goal is to investigate the impact of Basel regulation and, therefore, we implement the Basel Committee's recommendations.

All the conditions described above for VaR equally apply for treatments with ES. The only difference between VaR and ES is the implementation of the more stringent calculations of the daily minimum capital requirement when using ES. Therefore, all the equations and tables described above will be referenced for ES treatments when needed, as the trading algorithm is exactly the same.

3.2.2.2.3 Leverage Ratio

In our model, the capital measure consists of cash and we calculate the exposure measure using on-balance sheet exposures (stocks) and SFTs exposures (short-selling). As previously mentioned in section 2.1.5.3.1.3, the Basel Committee considers SFTs an important source of leverage and since short-selling is the sale of a borrowed security, the short-seller will have to buy the borrowed security back from the market in order to return it to the lender. Short-sellers can borrow securities in the repo or securities lending markets. Hence, even if the buy-back is not imposed in our model, we consider short-selling as a source of leverage. The buy-back is not mandatory as we do not build a network of borrowers and lenders, our agents buy and sell following their optimisation strategies regardless of whom their trading partners are. In the case of short-selling, an agent's strategy of shorting a position is matched by another agent's long strategy. We can interpret a potential continuous short-selling position as a succession of short-selling contracts with many different trading partners willing to enter this contract based on their optimisation strategies, and not necessarily an unrealistically extended short-selling contract beyond the settlement period.

The only mandatory buy-back, and consequently cancellation of the short position, is verified when the agent's optimisation strategy so determines, or when the regulatory treatment imposes a certain level for the LCR. When the calculation of the LR is smaller than the regulatory threshold of 3.25 percent, agents have to adopt measures to bring the ratio back above the threshold, which implies selling risky assets.

If in our model we had forced agents to buy-back (in treatments with short-selling) or pay-back (in treatments with leverage) it would have reduced the impact and significance of implementing both the LR and LCR regulatory framework. Therefore, agents don't payback nor buy-back, rather wait for their optimisation function or regulation to correct their positions.

The application of some rules is necessary to guarantee the correct implementation of capital requirements regulations. Hence, any regulation always overrides the result of the optimisation function of the agents, and both VaR and ES always override the result from LR and LCR regulations, as these are only supplementary regulatory measures.

The quarterly LR is updated at the end of each day and based on the average of the daily LR of the last 63 business days. The decision of buying or selling stocks depends on the calculation of the final amount of stocks, $q^i_{LR,t}$, which in turn, simultaneously, depends on and is used to calculate the quarterly LR, and LR^i_{t+1}, the future leverage ratio. Both these variables are a function of each other, directly or indirectly. The amount of stocks is constrained by the LR threshold of 3.25 percent, and the future leverage ratio depends on future funds and stocks, which are a function of the volume of the current order. This nonlinear problem involves complicated relations between the variables and this complex optimisation problem is solved by using DEPSO (Zhang and Xie [2003]). DEPSO is an algorithm for numerical optimisation problems and combines the advantages of Differential Evolution (DE) and Particle Swarm Optimization (PSO). Due to the randomisation used by DEPSO it becomes very likely to find the global optimum instead of getting trapped in local extrema. This non-linear optimisation method guarantees, *ceteris paribus*, that the future leverage ratio (LR^i_{t+1}) is minimised but remains above the threshold of 3.25 percent after the transaction completed while, simultaneously, maximises the portfolio optimisation function of the agent ($q^i_{LR,t}$). If the amount of risky assets is greater (smaller) than the amount of current stocks, than the agent should buy (sell), otherwise the agent should hold trading.

DEPSO numerical optimisation problem may be written as follows:

$$\begin{aligned}&\min f(x)\\&\text{subject to } g_i(x) =\in [c_i, d_i] \text{ for } i = 1, \ldots, n\end{aligned} \tag{3.32}$$

where $f(x)$ is the objective function, each $g(x)$ is a constraint function to be satisfied, and c and d are constants. Each function can be nonlinear and non-smooth. In the unconstrained optimisation problem $i = 0$ and, consequently, there is no constraint. Table 3.1 shows DEPSO setting parameters.

Algorithm 9 describes how DEPSO minimises LR^i_{t+1} and calculates $q^i_{LR,t}$, and how agents place orders based on these variables.

After implementing the risk-based capital requirements and the LR regulations, there is a smaller probability of placing an order directly derived from the

Algorithm 9 Portfolio optimisation trading algorithm with LR

Require: $q^i_{LR,t} \leftarrow [0, 100000] \wedge s^i_t \geq 0 \wedge c^i_t - k^i_t \geq 0$
Ensure: $min(LR^i_{t+1}) > 3.25\% \wedge q^i_{LR,t}$

```
begin DEPSO:
if q^i_{LR,t} > s^i_t then
    c^i_{t+1} ← c^i_t − q^i_{bLR,t} × b^i_t
    e^i_{t+1} ← s^i_t × p_t + q^i_{bLR,t} × b^i_t
else if q^i_{LR,t} < s^i_t then
    c^i_{t+1} ← c^i_t + q^i_{aLR,t} × a^i_t
    e^i_{t+1} ← s^i_t × p_t − q^i_{aLR,t} × a^i_t
else
    c^i_{t+1} ← c^i_t
    e^i_{t+1} ← s^i_t × p_t
end if
LR^i_{t+1} ← c^i_{t+1} / e^i_{t+1}
if e^i_{t+1} ← 0 then
    LR^i_{t+1} ← c^i_{t+1} / p_t
end if
output: LR^i_{t+1} ∧ q^i_{LR,t}
end DEPSO

if q^i_{LR,t} > s^i_t then
    q^i_{LR,t} ← q^i_{VaRLev,t}
else if q^i_{LR,t} < s^i_t then
    if decision ← buy then
        sell: q^i_{aLR,t} ← min(s^i_t − q^i_{aLR,t}, s^i_t); price ← max(b^i_t)
    else if decision ← q^i_{aVaRLev,t} then
        q^i_{aLR,t} ← s^i_t − q^i_{aLR,t}
        sell: q^i_{aLR,t} ← min(max(q^i_{aLR,t}, q^i_{aVaRLev,t}), s^i_t); price ← max(b^i_t)
    else if decision ← q^i_{aLev,t} then
        q^i_{aLR,t} ← s^i_t − q^i_{aLR,t}
        sell: q^i_{aLR,t} ← min(s^i_t − q^i_{aLR,t}, s^i_t); price ← max(b^i_t)
    else if decision ← hold then
        sell: q^i_{aLR,t} ← min(s^i_t − q^i_{aLR,t}, s^i_t)
    else
        hold
    end if
else if q^i_{LR,t} = s^i_t then
    if decision ← q^i_{aLev,t} then
        sell: q^i_{aLR,t} ← min(s^i_t − q^i_{aLR,t}, s^i_t); price ← a^i_t
    else if decision ← q^i_{aVaRLev,t} then
        sell: q^i_{aLR,t} ← q^i_{aVaRLev,t}; price ← max(b^i_t)
    else
        hold
    end if
end if
```

Table 3.1: DEPSO setting parameters

Description	Value
Number of agents	70
Maximum learning cycles	200
Constraint-handling	Basic constraint-handling
DE: scale constant	0.5
DE: crossover constant	0.9
PSO: learning factor for pbest	1.494
PSO: learning factor for gbest	1.494
PSO: inertia weight	0.729

Note: Zhang and Xie [2003]

original risk management optimisation strategy, as both regulatory constraints can override the agents' portfolio optimisation decision.

3.2.2.2.4 Liquidity Coverage Ratio

In our model, cash is considered HQLA and short-selling and leveraged positions are considered future cash outflows due to interbank lending, even if these exposures are not explicitly represented in the model in the form of a network of dependencies. Naked short-selling happens when investors sell securities that they do not possess and have not confirmed their ability to possess in the future. This practice is possible in our model and we consider it temporary or unintentional rather than market manipulation.

Under the regulatory framework agents have to buy the necessary risky assets to maintain the LCR ratio above the threshold of 100 percent. Only if the LCR is below this threshold agents have to implement algorithm 10, otherwise the amount calculated under risk-based capital treatments regulation or portfolio optimisation is used to place an order.

3.2.2.2.5 Leverage Ratio and Liquidity Coverage Ratio

The realistic scenario is the existence of both leverage and short-selling. Therefore in this section we present the cases were the LR and LCR regulations are

Algorithm 10 Portfolio optimisation trading algorithm with LCR

Require: $q^i_{LCR,t} \leftarrow [s^i_t, 100000] \wedge c^i_t - k^i_t \geq 0$
Ensure: $min(LCR_{t+1})^i > 100\% \wedge q^i_{LCR,t}$
  begin DEPSO:
  if $q^i_{LCR,t} > s^i_t$ **then**
    $hqla^i_{t+1} \leftarrow c^i_t - q^i_{bLCR,t} \times b^i_t$
    $nco^i_{t+1} \leftarrow |s^i_t \times p_t + q^i_{bLCR,t} \times b^i_t|$
  else if $q^i_{LCR,t} < s^i_t$ **then**
    $hqla^i_{t+1} \leftarrow c^i_t + q^i_{aLCR,t} \times a^i_t$
    $nco^i_{t+1} \leftarrow |s^i_t \times p_t - q^i_{aLCR,t} \times a^i_t|$
  else
    $hqla^i_{t+1} \leftarrow c^i_t$
    $nco^i_{t+1} \leftarrow |s^i_t \times p_t|$
  end if
  $LCR^i_{t+1} \leftarrow \frac{hqla^i_{t+1}}{nco^i_{t+1}}$
  if $nco^i_{t+1} \leftarrow 0$ **then**
    $LCR^i_{t+1} \leftarrow \frac{hqla^i_{t+1}}{p_t}$
  end if
  output: $LCR^i_{t+1} \wedge q^i_{LCR,t}$
  end DEPSO

  if $q^i_{LCR,t} < s^i_t$ **then**
    $q^{i*}_{LCR,t} \leftarrow q^i_{VaRSS,t}$
  else if $q^i_{LCR,t} > s^i_t$ **then**
    if decision $\leftarrow$ buy **then**
      buy: $q^i_{bLCR,t} \leftarrow |s^i_t - q^i_{LCR,t}| \Rightarrow q^i_{bLCR,t} \leftarrow min(q^i_{bLCR,t}, q^i_{bVaRSS,t})$; price $\leftarrow b^i_t$
    else if decision $\leftarrow$ (sell $\vee$ hold) **then**
      if decision $\leftarrow$ sell **then**
        if $k^i_t < c^i_t$ **then**
          buy: $q^i_{bLCR,t} \leftarrow |s^i_t - q^i_{LCR,t}| \Rightarrow q^i_{bLCR,t} \leftarrow min(q^i_{bLCR,t}, q^i_{bVaRSS,t})$; price $\leftarrow b^i_t$
        else if $k^i_t > c^i_t$ **then**
          sell: $q^i_{aLCR,t} \leftarrow q^i_{aVaRSS,t}$; price$\leftarrow max(b_t)$
        end if
      else if $q^i_{VaRSS,t} \leftarrow$ hold **then**
        if $\frac{c^i_t}{b^i_t} < 1 \vee \frac{c^i_t - k^i_t}{b^i_t} < 1$ **then**
          hold
        else
          buy: $q^i_{bLCR,t} \leftarrow min(|s^i_t - q^i_{bLCR,t}|, \frac{c^i_t}{b^i_t})$; price$\leftarrow b^i_t$
        end if
      end if
    end if
  else if $q^i_{LCR,t} = s^i_t$ **then**
    if decision $\leftarrow q^i_{aSS,t}$ **then**
      sell: $q^i_{aLCR,t} \leftarrow s^i_t - q^i_{aSS,t}$; price $\leftarrow a^i_t$
    else if decision $\leftarrow q^i_{aVaRSS,t}$ **then**
      sell: $q^i_{aLCR,t} \leftarrow q^i_{aVaRSS,t}$; price $\leftarrow max(b^i_t)$
    else
      hold
    end if
  end if

applicable and the respective algorithm.

Table 3.2 shows the cases where leverage and liquidity are considered and investigated in our model and, consequently, when the LR and LCR are applicable.

Table 3.2: Leverage and Liquidity

Cash Position	Stock Position	Applicable Regulation	Applied Regulation
+	+	LR	LR
+	-	LR and LCR	LCR
-	+	LR and LCR	LR
-	-	LR and LCR	N/A

Note: Negative cash positions represent borrowing and negative stock positions represent short-selling. When both positions are negative, or when total wealth is negative, it is assumed that agents are in technical default.

When both amounts of cash and stocks are negative agents are considered to be in technical default, which disallow them to actively participate in the market until at least one of these components become positive and greater than the other (negative) component. If one of the positions, cash or stocks, is positive it might still be the case of technical default if the negative position is greater than the positive one, which results in a negative wealth position (v. equation 3.5) and subsequently technical default.

If both cash and stocks are positive there is no case to apply the LCR, as short-selling is only relevant for liquidity purposes, which is not applicable in this case. However, regulation on leverage ratios is implemented under this scenario as stocks are agents' exposure.

In the scenario where only leverage is present, the cash position is negative and the stocks position is positive, both LR and LCR are negatively affected. Hence, both LR and LCR are applicable. However, implementing the LR trading algorithm, which brings the LR above the threshold of 3.25 percent, simultaneously eliminates the violation of the LCR since both cash and stocks become positive. Interbank lending maturity is irrelevant in our model, as we are only interested in the analysis of the LR and its position above 3.25 percent. Hence, the network of dependencies between agents becomes irrelevant. For example, if an agent maintains a leveraged position for long periods of time, it doesn't necessarily reflect an unrealistic long interbank lending maturity. It might be the case of an agent continuously borrowing from different lenders.

Lastly, in the case where the amount of cash is positive and the amount of stocks is negative, both LR and LCR are applicable due to short-selling. As in the previous case, despite this scenario being covered by both regulatory frameworks, the implementation of just one of them simultaneously solves the other violation. In this case the application of the LCR also eliminates the LR violation. Short-selling is assumed to be less than 30 days maturity, therefore all short-selling positions are considered in terms of LCR calculation. The amount of cash is never negative in this scenario, as that particular case is captured and solved under the LR regulation.

The algorithm 11 reflects the treatment where the LR and LCR are applied. If both ratios are above the minimum required threshold this algorithm is not applicable. However, if either the minimum required threshold for LR or LCR, or both, are violated then the respective algorithm is called and used by the agents to maximise their positions under these constraints.

The agents' risk management strategies are influenced by the introduction of different regulatory frameworks, which are applied in our model as a sequence of regulatory layers that limit the agents' trading behaviour. We can briefly summarise the general trading rules as follows: 1) Any regulatory constraint overrides the agents' initial portfolio maximisation decision, and 2) the risk-based capital requirement constraints, VaR and ES, override the leverage and liquidity ratios. For example, if the initial portfolio optimisation defines as optimal trading direction buying a certain amount of stocks, which is then overridden by the risk-based capital requirements constraint, VaR or ES, determining that the agent should sell rather than buy, even if the LCR is above the minimum required threshold and indicates buying as optimal, the capital requirements constraint prevails over the other two strategies and the agent places a sell order in the order-book.

Algorithm 11 Portfolio optimisation trading algorithm with LR and LCR

Require: $q^i_{LCR,t} \leftarrow [s^i_t, 100000] \wedge q^i_{LR,t} \leftarrow [0, 100000] \wedge c^i_t - k^i_t \geq 0$
Ensure: $min(LCR^i_{t+1}) > 100\% \wedge min(LR_{t+1})^i > 3.25\% \wedge q^i_{LRLCR,t}$
  if $LR \geq 0.0325 \wedge LCR \geq 1$ **then**
    $q^i_{LRLCR,t} \leftarrow q^i_{VaRLevSS,t}$
  else if $LR < 0.0325 \wedge s^i_t > 0$ **then**
    $LRLCR^i_{t+1} \leftarrow LR^i_{t+1}$
    if $q^i_{LRLCR,t} > s^i_t$ **then**
      $q^i_{LRLCR,t} \leftarrow q^i_{bVaRLevSS,t}$
    else if $q^i_{LRLCR,t} < s^i_t$ **then**
      if decision $\leftarrow$ buy **then**
        buy: $q^i_{aLRLCR,t} \leftarrow |s^i_t - q^i_{LRLCR,t}|$; price $\leftarrow max(b_t)$
      else if decision $\leftarrow$ *sell* $\wedge k^i_t > c^i_t$ **then**
        sell: $q^i_{aLRLCR,t} \leftarrow max(q^i_{aLRLCR,t}, q^i_{aVaRLevSS,t})$; price $\leftarrow max(b_t)$
      else if decision $\leftarrow$ *sell* $\wedge k^i_t \leq c^i_t$ **then**
        sell: $q^i_{aLRLCR,t} \leftarrow |s^i_t, q^i_{aLRLCR,t}|$; price $\leftarrow max(b_t)$
      else if decision $\leftarrow$ hold **then**
        sell: $q^i_{aLRLCR,t} \leftarrow |s^i_t, q^i_{aLRLCR,t}|$; price $\leftarrow max(b_t)$
      end if
    else if $q^i_{LRLCR,t} = s^i_t$ **then**
      if decision $\leftarrow$ *sell* $\wedge k^i_t \leq c^i_t$ **then**
        sell: $q^i_{aLRLCR,t} \leftarrow s^i_t - q^i_{aLevSS,t}$; price $\leftarrow a^i_t$
      else if decision $\leftarrow$ *sell* $\wedge k^i_t > c^i_t$ **then**
        sell: $q^i_{aLRLCR,t} \leftarrow q^i_{aVaRLevSS,t}$; price $\leftarrow max(b_t)$
      end if
    end if
  else if $LCR < 1 \wedge s^i_t \leq 0$ **then**
    if $q^i_{LRLCR,t} > s^i_t$ **then**
      if decision $\leftarrow$ buy **then**
        buy: $q^i_{bLRLCR,t} \leftarrow |s^i_t - q^i_{LRLCR,t}| \Rightarrow q^i_{bLRLCR,t} \leftarrow min(q^i_{bLRLCR,t}, q^i_{bVaRLevSS,t})$; price $\leftarrow b^i_t$
      else if decision $\leftarrow$ *sell* $\vee$ *hold* **then**
        if decision $\leftarrow$ *sell* $\wedge k^i_t < c^i_t$ **then**
          buy: $q^i_{bLRLCR,t} \leftarrow |s^i_t - q^i_{LRLCR,t}| \Rightarrow q^i_{bLRLCR,t} \leftarrow min(q^i_{bLRLCR,t}, q^i_{bVaRLevSS,t})$; price $\leftarrow b^i_t$
        else if decision $\leftarrow$ *sell* $\wedge k^i_t > c^i_t$ **then**
          sell: $q^i_{aLRLCR,t} \leftarrow q^i_{aVaRLevSS,t}$; price $\leftarrow max(b_t)$
        else if decision $\leftarrow$ hold **then**
          buy: $q^i_{bLRLCR,t} \leftarrow |s^i_t - q^i_{LRLCR,t}|$; price $\leftarrow b^i_t$
        else if $q^i_{LRLCR,t} < s^i_t$ **then**
          $q^i_{LRLCR,t} \leftarrow q^i_{VaRLevSS,t}$
        else if $q^i_{LRLCR,t} = s^i_t$ **then**
          if decision $\leftarrow$ *sell* $\wedge k^i_t \leq c^i_t$ **then**
            sell: $q^i_{aLRLCR,t} \leftarrow q^i_{aLevSS,t}$; price $\leftarrow a^i_t$
          else if decision $\leftarrow$ *sell* $\wedge k^i_t > c^i_t$ **then**
            sell: $q^i_{aLRLCR,t} \leftarrow q^i_{aVaRLevSS,t}$; price $\leftarrow max(b_t)$
          else
            hold
          end if
        end if
      end if
    end if
  else
    $q^i_{LRLCR,t} \leftarrow q^i_{VaRLevSS,t}$
  end if

3.3 Parameters Settings

In this section we describe in detail all the parameters settings for all treatments. The results reported per treatment are the outcome of 100 simulations of 504 business days each, equivalent to a period of 2 years, with different seed values for the pseudo-random number generator. We allow for a transitory regime and discard the first 10 days (each simulation runs over 514 days) in order for the model to reach a stable behaviour after the usual initial instability. The model stabilises relatively quickly, thus the 10-days margin is sufficient for our data and consequent analysis not to be affected in case of any exceptional slower stabilisation process. Each day consists of 510 equally-spaced discrete time steps, representing 1 minute intervals over a period of 8.5 hours of a continuous trading session, for example on the main order-book of the London Stock Exchange, which represents 257040 time-steps per simulation. We repeat the simulations within each treatment varying the initial conditions to test the robustness of the results to sensitivity.

Table 3.3 summarises the initial conditions and dependent variables of the model common to all experimental treatment conditions. Table 3.4 shows the initial conditions that differ from M-V treatments to CPT treatments. The initial conditions and dependent variables exclusively used in treatments with VaR and ES are identified separately in table 3.5. Tables 3.6 and 3.7 summarise the initial conditions of the experiments with LR and LCR, respectively.

3.3.1 Parameters

At the end of each day, financial institutions earn interest equal to the risk-free rate, r_f. The overnight risk-free rate was calculated based on the Bank of England annual average of the sterling overnight index average (SONIA) lending rate between the years 2001-2017, 4.47 percent, and using the following day count convention:

$$\frac{\text{Number of days between dates}}{\text{Number of days in reference period}} \times \text{Interest in reference period} \tag{3.33}$$

The probability of arrival at the market, λ^i, is a function of the weight of the

Table 3.3: Initial conditions and distributions for dependent-variables for all treatments

Simulation	
Runs	100
Days	504
Length of days	510
Market	
Yearly risk-free rate: r_f	4.47%
Arrival rate: λ^i	$1 -$ proportion of fundamentalist component
Initial price: p_0	$\sim U(400, 900)$
Fundamental Price: p_t^f	
GBM drift: μ	2.07×10^{-7}
GBM volatility: σ	3.97×10^{-4}
GBM Wiener process: ϵ	$\sim N(0, 1)$
Population	
Population size: n_a	$\sim U(500, 5000)$
Maximum chartist horizon:L_{max}	85
Chartist window interval size	30
Proportion of institutional agents	50%
Power Law Exponent: κ^i	$\sim Exp(10)$
Initial cash: c_0^i	$\sim U(50000, 100000)$
Initial stock: s_0^i	$\sim U(50, 200)$
Time horizon: τ^i	$\tau(\lambda^i)$
Agents' components weights	
Fundamentalist: g_1^i	$\sim N(0, \sigma_1)$
Chartist: g_2^i	$\sim N(0, \sigma_2)$
Noise: n^i	$\sim N(0, n_0)$
σ_1	$\sim U(0.1, 1.0)$
σ_2	$\sim U(0.1, 1.4)$
n_0	$\sim U(0.001, 0.009)$

Table 3.4: Initial conditions and distributions for dependent-variables that differ between M-V and CPT

Population M-V	
Risk aversion: A^i	$\sim N(3, 0.5)$
Population CPT	
Loss aversion: λ^i	$\sim U(1.5, 3)$
Risk aversion - gains: α^i	$\sim U(0.875, 0.905)$
Risk aversion - losses: β^i	$\sim U(0.905, 0.935)$
Exponent for gains: γ^i	$\sim U(0.28, 1)$
Exponent for losses: δ^i	$\sim U(0.28, 1)$

Table 3.5: Initial conditions and distributions for dependent-variables for risk-based capital requirements treatment

Market	
Arrival rate: $\lambda^i_{VaR,t}$, $\lambda^i_{ES,t}$	*if* $(k^i_{VaR,t}, k^i_{ES,t}) > c^i_t$ *then* 0.025 *otherwise* λ^i
Regulation	
VaR horizon	252 days
sVaR horizon	504 days
Average VaR horizon	60 days
VaR confidence interval	99%
ES confidence interval	97.5%
Minimum holding period	10 days
Multiplication Factors: m_c, m_s	$[3, 4]$
Backtesting	
VaR backtesting horizon	252 days
sVaR backtesting horizon	504 days
Confidence interval	99%
Green zone	$[0, 0.95[$
Yellow zone	$[0.95, 0.9999[$
Red zone	$[0.9999, 1]$

Table 3.6: Initial conditions and distributions for dependent-variables for leverage ratio treatment

Market	
Arrival rate: $\lambda^i_{LR,t}$	if $LR \geq 3.25\%$ then λ^i otherwise 0.025
Regulation	
Maximum leverage	60
LR	$\geq 3.25\%$

Table 3.7: Initial conditions and distributions for dependent-variables for liquidity coverage ratio treatment

Market	
Arrival rate: $\lambda^i_{LCR,t}$	if $LCR \geq 100\%$ then λ^i *otherwise* 0.025
Regulation	
Maximum short-selling	-60
LCR	$\geq 100\%$

fundamentalist component of each agent. These parameters are initially randomly assigned across agents. The greater (lower) this weight the smaller (bigger) is the probability of that agent entrying the market, as chartist and noise trading is characterised by intraday activity contrary to longer periods of inactivity of fundamentalist traders. The agents' components weights, g_1^i, g_2^i and n^i (v. equation 3.2), are randomly assigned across agents at the beginning of each simulation.

Chiarella and Iori [2002] define the mean of the weight of the fundamentalist, chartist and noise components as 0 and set the standard deviation as 1, 1.4 and 3, respectively. We opt not to set these parameters selectively. Historical stock prices daily volatility in the United Kingdom is, on average, approximately 1 percent[4]. Hence, in order to choose approximate values to those set by Chiarella and Iori [2002] and, simultaneously, obtain a daily volatility of approximately 1 percent, we set these parameters to $\sigma_1 \sim U(0.1, 1.0)$, $\sigma_2 \sim U(0.1, 1.4)$ and $n_0 \sim U(0.001, 0.009)$. We choose the parameters set in Chiarella and Iori [2002] as the maximum of the uniform distribution for the fundamentalist and chartist components, and we defined the interval for the weight of the noise component to obtain an average annual volatility of approximately 16 percent[5].

The parameters of the distribution of the initial price are set taking into consideration the parameters of the initial amounts of cash and stocks. The initial amounts of cash and stocks are chosen with the objective of setting initial proportions of cash and stocks in agents' portfolio representing approximately 50 percent of the agents' total wealth. This setting permits analysing the evolution of portfolio dediversification from a relatively balanced initial position rather than from portfolios with initial conditions biased towards more cash or more risky asset, which could influence agents' portfolio diversification. Therefore, we define $c_0^i \sim U(50000, 100000)$ and $s_0^i \sim U(50, 200)$ as distributions for initial cash and stocks, respectively. Given these intervals, the setting of the initial price should not be either too low, to avoid too many orders placed with a similar

[4]World Bank, Volatility of Stock Price Index for United Kingdom [DDSM01GBA066NWDB], retrieved from FRED, Federal Reserve Bank of St. Louis, `https://fred.stlouisfed.org/series/DDSM01GBA066NWDB`, March 14, 2018.

[5]This value is just indicative. The average annual volatility in our model varies depending on the experimental treatment. For example, the average annual volatility in the experimental treatment with LR and LCR is 95.07 percent and 65.79 percent for VaR and ES treatments, respectively. However it is very small to all baseline treatments.

price, which would eliminate most of the volatility in the market, or too high, since it would reduce the number of transactions and respective volume, also affecting the behaviour of the order-book. Therefore, the initial price was set as $p_0 \sim U(400, 900)$.

In theory, the fundamental price of a stock is determined by the present value of expected future earnings that accrue to shareholders. Hence, we use a historical sample of stock market real earnings data[6], from January 1871 to September 2017, to define the trajectory of the fundamental price in time. This data set consists of monthly data that was converted to tick data to allow the calculation of the fundamental price per tick, or time step. In our model, the fundamental price follows a GBM (v. equation 3.1) and the parameters μ and σ are set by using, respectively, the mean and standard deviation of the growth rate of the real earnings defined as follows:

$$GRE_t = \frac{RE_t - RE_{t-1}}{RE_{t-1}} \tag{3.34}$$

where RE_t represents the real earnings at time t. Real earnings at time $t-i$ are defined as

$$RE_{t-i} = \frac{E_{t-i} \times CPI_t}{CPI_{t-i}} \tag{3.35}$$

where E_{t-i} represents the earnings at time $t-i$, and CPI is the Consumer Price Index. We opt for earnings rather than dividends as firms may decide not to pay dividends and instead repurchase shares as a way of returning earnings to shareholders (Campbell and Shiller [1988]). Hence, we use a valuation model (Fama and French [2002] and Boswijk et al. [2007]) based on past observation of earnings to define the trajectory of fundamental prices.

The population size follows a uniform distribution with a sufficiently large interval to allow for heterogeneous population sizes. Other authors, for example Chiarella and Iori [2002], Chiarella et al. [2009] and Hermsen [2010], opted for a fixed number of agents, from 500 to 5000 agents. As in other parameters of the model, we rule out a discretionary approach and instead draw a range of values

[6]Shiller, R., Online data U.S. Stock Markets 1871-Present, retrieved from Yale University, Department of Economics; http://www.econ.yale.edu/ shiller/data.htm, March 15, 2018.

from a uniform distribution with interval $[500, 5000]$ to test model sensitivity and robustness to initial conditions.

The maximum horizon, L_{max}, of the chartist rule, $\overline{r}_{t,L^i}$ (v. equation 3.3), is set to a weekly window with a maximum of 85 periods, each of which consists of 30 time steps. Thus, the chartist rule determines a maximum horizon corresponding to 2550 time steps, the equivalent to one trading week in our model. The *rationale* behind the implementation of chartist windows over a maximum of a trading week as the basis for the chartist rule derives, firstly, from the fact that the influence of technical analysis on traders is particularly relevant at the shortest horizons, e.g. intraday to one week (Taylor and Allen [1992]), and, secondly, from the concept of *refresh time* sampling (Barndorff-Nielsen et al. [2011]). Non-synchronous trading delivers fresh prices at irregularly spaced times. The chartist windows are used by financial institutions in our model to look back at past prices and use them to derive price statistics subsequently used in the chartist rule of the expected returns (v. equation 3.3) and in the mean-variance portfolio optimisation. If the observation of past prices reveals only, or mostly, stale prices, as it can be the case if there are no transactions updating the current price, under these conditions the chartist rule would not be of any use for the financial institution since the difference between successive prices would be 0.

The percentage of institutional agents in the model assumes particular importance as it may play an important role in stock market liquidity (Blume and Keim [2012]). Institutional agents are the only ones affected by the regulatory constraint and high percentages of this type of agent may lead to an order-book depletion on its buy side due to the necessity of fulfilling the minimum capital requirements by selling risky assets. Hence, it becomes extremely important to use a proportion of institutional agents taken from the empirical evidence in the stock markets to avoid biased results due to unrealistic orders and subsequently an unbalanced order-book. The proportion of equities managed by institutional investors started to increase during the second half of the 20^{th} century, reaching 67 percent of the value of all stocks by the end of 2010 in the US (Blume and Keim [2012]). In 2007, the private institutional owners owned 44 percent of the value of the London Stock Exchange (of Trade Industry and Fisheries [2014]). Hence, the bottom line to our approach is that institutional investors own or manage a large share of the equity market. The institutional ownership in our simulation is 50 percent, which is represented by a proportion of 50 percent of institutional agents. Notwithstanding the share of institutional agents in the market is not necessarily equivalent to the share of stocks traded, we can reasonably assume that the share of the institutional agents' stocks value is not far from 50 percent due to, firstly, the uniform statistical distribution determining the assignment of the initial endowment of stock, and, secondly, to the binomial distribution used to assign an institutional type to agents.

The power law exponents κ^i (v. equations 3.6 and 3.7) are randomly assigned across financial institutions using a negative exponential distribution with a rate parameter of 10. This power-law behaviour of the limit prices makes traders place their orders relatively far away from the best price, as pointed out in Zovko and Farmer [2002] and Bouchaud et al. [2002] based on empirical data. Despite the fact that roughly half of the total number of orders exhibit a small difference between the limit price and the best price, in some cases the distribution of limit orders prices could be 50 percent above or below the stock price (Bouchaud et al. [2002]). Financial institutions believe that very large variations of the price are possible within a short time horizon. These beliefs might be explained, for example, by traders' over-optimistic expectations about the executions of orders far from the current price or mistakes made by traders while forming expectations. It is important to stress the fact that, both in real markets (Zovko and Farmer [2002]) and in our model, an increase in the average difference between the limit price and the best price available will lower the limit-order book depth. Although, Zovko and Farmer [2002] conclude that volatility influences the distance between the limit price and the best price, at least partially. These authors also suggest that traders probably use volatility as a signal when placing orders, which supports the credible hypothesis that traders are reasonably aware of the volatility distribution when placing orders. In our simulation, financial institutions incorporate historical volatility in the optimal proportion of the risky asset in the mean-variance optimisation strategy (v. chapter 5), and in the limit of the integral of the value function of the CPT-agents (v. chapter 6).

In a model where orders are formed using the expected price, which depends on the expected return (v. equation 3.4), rather than the bid/ask prices, the power law exponents with mean of 0.1 help keeping orders close to the best prices. If the exponent introduces more diffusion, this effect would increase the uncertainty of errors originated from the expected future price. Hence, the exponential distribution with rate parameter 10 stabilises the price around the bid/ask while simultaneously allows orders far from the best prices to occasionally emerge.

Typical risk aversion coefficients, A^i, for the representative investor range from 2.0 through 4.0 (Friend and Blume [1975]), with the latter representing less tolerance to risk. In our model, financial institutions are assigned an initial level of risk aversion using a normal distribution with mean 3 and variance 0.5.

Agents' time horizon, τ^i, is represented as a function of the probability of arrival at the market $(t + \tau(\lambda^i))$. Only when an agent enters the market an order can be placed in the order-book.

Table 3.4 shows the parameters exclusively used in CPT simulations. The CPT value function curve is concave above the reference point and convex below if both α and β (v. equation 2.15), parameters of risk aversion, are less than 1. Empirical analysis suggests that β is greater than α and that both parameters need to be close to each other to generate investment proportions in risky and risk-free asset close to those observed in some markets (Davies and Satchell [2004]). Abdellaoui [2000] calculates median estimates of α and β of 0.89 and 0.92, respectively, assuming a piecewise power utility function as in Tversky and Kahneman [1992]. These estimates are close to Tversky and Kahneman [1992] estimates of 0.88 for both α and β, and are consistent with the hypothesis of diminishing sensitivity, as α and β capture diminish sensitivity to increases in absolute payoffs $(0 < \alpha, \beta < 1)$. The impact of a change in the absolute payoff decreases with the distance from 0 (Tversky and Kahneman [1992]). According to Tversky and Kahneman [1992], the median of the loss aversion (λ) was 2.25, which is in the center of the interval of the uniform distribution used in our model to calculated λ^i.

Our model is based on the original piecewise power utility function (Tversky and Kahneman [1992]). However, it has been noted that this specific form of the value function violates loss aversion (Kobberling and Wakker [2005], De Giorgi et al. [2011]), unless $\alpha = \beta$ and $\lambda > 1$. These authors suggest a piecewise CARA value function based on exponential utilities. However, He and Zhou [2011] note that this solution does not avoid potential problems.

Loss aversion is violated for small deviations from the reference point when $\alpha < \beta$ (Bernard and Ghossoub [2010]). In our implementation $\alpha < \beta$ and $\lambda > 1$, so our model might violate loss aversion for small deviations from the reference point, according to Bernard and Ghossoub [2010]. The interval on which loss aversion is violated depends on the distance between α and β. In our model α and β are very close, hence the interval is infinitesimal (Bernard and Ghossoub [2010]):

$$0 < x < \epsilon = \lambda^{\frac{1}{\alpha-\beta}}$$

Nevertheless, Baucells and Heukamp [2006] show empirical evidence supporting violations of loss aversion when individuals use the probability of strictly positive gains as a heuristic, in particular with small stakes. Therefore, there is some empirical evidence supporting the hypothesis $\alpha < \beta$, which also has the benefit to allow the use of a closed-form solution for the calculation of the optimal holding of the risky asset, as we will see in section 6.1.

Parameters γ and δ describe the amount of over- and underweighting in the weighting functions. Assuming the parameters of the weighting functions, γ and δ (v. equation 2.16), are greater than 0.28 ensures that both functions, for gains and losses, are increasing (Barberis and Huang [2008]). These authors consider γ and δ in the interval $0.28 < \gamma, \delta < 1$. The curvature of the weighting functions reflects the overweighting of small probabilities and the underweighting of high probabilities, both for positive and negative prospects, and further enhance the risk aversion for gains and risk seeking for losses. Tversky and Kahneman [1992] estimate the median values for γ and δ as 0.61 and 0.69, respectively. Abdellaoui [2000] and Abdellaoui et al. [2007] estimate approximate values, $\gamma = 0.6$ and $\delta = 0.7$, and Wu and Gonzalez [1996] estimate $\delta = 0.71$. Hence, the distributions of the parameters in our model follow the empirical evidence.

The initial conditions in tables 3.5–3.7 emanate from the regulations and were previously covered in section 2.1.5.3, except the arrival rate and the maximum levels of leverage and short-selling. If the regulatory constraints are not applicable, then the agents' arrival rate is the same as in the baseline treatments and only depends on the parameter λ^i. However, when at least one of the financial regulations constraints is verified, then institutional agents are recurrently called to enter the market and the rate of arrival is 2.5 percent. This process guarantees that all institutional agents violating at least one of the financial regulations constraints are called and the probability of entering the market is 2.5 percent. This probability is small enough to avoid flooding the market with mandatory and simultaneous institutional agents' selling market orders. For example, when institutional agents violate the VaR condition at the end of day t, in day $t+1$ they will be recurrently called to enter the market in every time step, thus for a maximum of 510 times during the day, until their position is within the threshold imposed by the regulations. The probability of entering the market is no longer the share of their fundamentalist component but a small and constant probability of 2.5 percent to avoid all institutional agents currently violating the VaR condition entering the market in the same moment and with that flooding the market with their short positions. These conditions allow institutional agents to correct their positions and fulfil the regulatory requirements during the next business day and, simultaneously, avoid the artificial instability that would be created by a disproportionate number of selling order placed simultaneously.

The maximum level of leverage and short-selling of 60 was determined based on the maximum level of leverage UK banks exhibit before the crisis of 2007-2008 (BoE [2012]). The leverage reached its peak during the crisis and UK banks maximum leverage observed was above 60 times capital. UK banks median average between 1960-2011 has been 20-30 times capital (BoE [2012]), therefore we choose a relatively high level of leverage to capture extreme behaviours similar to the ones observed during the financial crisis of 2007-2008. The level of short-selling replicates the maximum level of leverage but with negative sign.

3.4 Conclusion

Our hypothesis and research questions are tested using an ABM which experimental design is divided in treatments with and without leverage and short-selling, EUT and CPT, and Basel III regulation. This model allows the identification of potential impacts on banks' behaviour and market stability of different market conditions and optimisation strategies and the implementation of financial regulations.

The market structure and economic agents' behaviour are suitable for investigating the implications of the financial regulations and most of the parameters settings in the model were derived from empirical evidence.

This chapter covered the general features of the model. The agents' strategy-specific features, which are distinguishable between optimisation strategies, are covered in the respective chapters 5 and 6.

Lastly, we make all the code available upon request.